
# Fixed Costs and Irreversibility

Investment, Finance and Asset Prices ECON5068

**Thomas Walsh**

Adam Smith Business School

# Lecture Overview

- **Motivation - Inaction**
- **Evidence**
- **Fixed Costs** of Adjusting
- **Irreversibility**/Specificity of Installation and "Stranded Assets"
- Reading:
    1. Cooper and Haltiwanger (2006),
       On the Nature of Capital Adjustment Costs,
       Review of Economic Studies.

# Other Costs of Investing

- key assumption in Q model is that **adjustment costs are convex**.
- For example, $AC_t = \frac{\phi}{2} I_t^2$, a quadratic function is a common assumption.
- This assumption implies that the **investment rule is continuous and smooth**
- **Costs change smoothly with the size of investment**
  1. Since the adjustment function is convex, firms want to avoid large changes in investment as the cost rises at an increasing rate.

- **Is this the only cost faced by firms when growing their asset base?**

# Evidence in the data

## Costs Firms Face

- Think about investment at a factory or plant, expanding capacity
- In addition to the standard convex cost (a growing pains/tiredness proxy), firms/plants face:

1. Some projects are **very hard to get started**. Fixed cost of any action
2. **Disruption** costs from replacing existing capital.
3. Costly learning as the structure of production might have changed.
4. Delivery lags and time to build delays.
5. **Irreversibility** of projects due to lack of secondary market for capital goods. Once installed, there is no turning back.

# Lumpiness - Evidence from Data

- Micro evidence from plant-level data indicate investment is **lumpy**.

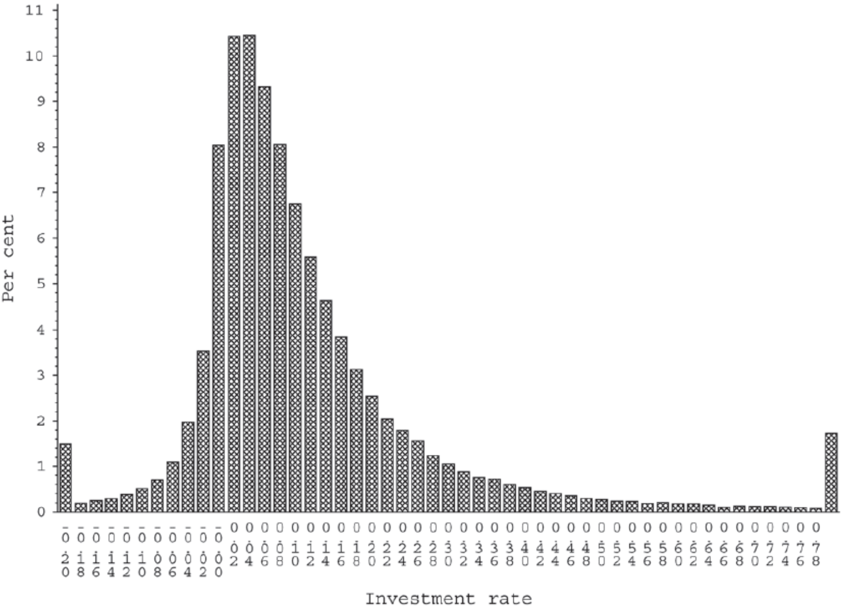
**Investment is LUMPY**

Firm-level investment is characterised by

- Long periods of inaction ($I \approx 0$)
- Bursts of large, infrequent investment activity.
- These spells can be several years, once started
- Lumpy: Not Continuous, not smooth. Clustered or bunched activity.

- This evidence suggests **convex adjustment costs may not be the whole story** in terms of capital adjustment.

# Investment Rate (I/K) Distribution



# Capital Adjustment: Cooper & Haltiwanger (2006)

**Table 1:** Summary Statistics: U.S. Manufacturing Plant-Level Investment

| Variable | Mean | (Std. Dev.) |
|---|---|---|
| **Investment Activity** | | |
| Average investment rate | 12.2% | (0.10) |
| Inaction rate (abs($I/K$) $< 1\%$) | 8.1% | (0.08) |
| Negative investment rate | 10.4% | (0.09) |
| **Lumpy Adjustment** | | |
| Spike rate: positive, $(I/K) > 20\%$ | 18.6% | (0.12) |
| Spike rate: negative, $(I/K) < -20\%$ | 1.8% | (0.04) |
| **Dynamics** | | |
| Serial correlation of investment rates | 0.058 | (0.003) |
| Correlation: profit shocks & investment | 0.143 | (0.003) |

Source: Longitudinal Research Database (LRD), U.S. manufacturing plants

# Nonconvex Costs - Evidence from Data

- Source: Longitudinal Research Database
- 7000 large manufacturing plants that were continually in operation betweeen
- Years: 1972 and 1988.
- This is the highest level of disaggregation, since a firm is a collection of plants (factories/sites) and branches.
- First, notice the shape of the distribution: not symmetric, rather skewed to the right
- If these data were generated from a model with convex adjustment cost $\Rightarrow$ distribution would be symmetric/ (double) bell shaped.

## Nonconvex Costs - Evidence from Data

- Second, there are periods of investment inaction, 8% of observations, plants do nothing.

- Again, with convex adjustment costs only there will be no periods of inaction
  1. Plant will be doing a little bit of investment in all periods.

- Third, there is asymmetry: 80% of observations entail positive investment rate, while only 10% of observations entail negative investment rate
  1. Suggests firms really dislike selling capital.

# Nonconvex Costs - Modelling Non-Convex Costs

- The message from this data is that firms likely face more than just convex adjustment costs.
- Firms face **irreversibility** of investments - "the toothpaste cannot go back in the tube / the toast cannot be unburnt"
- **Fixed costs** of adjustment (in combination with convex adjustment costs) can explain these patterns of adjustment much better than convex costs alone. "In for a penny, in for a pound"; Switching modes / "Getting started is hard"

# Fixed Costs and the Lumpy Adjustment Model

# Non Convex Costs

- Workhorse model of firm investment
- **Cooper and Haltiwanger** consider a dynamic programming problem specified at the plant level as:

$$V(A,K,p) = \max\left\{V^i(A,K,p), V^a(A,K,p)\right\} \quad \text{for all } (A,K,p) \tag{1}$$

- where $V^a$ is the value of the firm when it invests actively (a for **adjust**)
- $V^i$ is the value of **inactivity** (no investments).
- The optimal value of the firm is the **maximum** of these two options.

## Nonconvex Costs

- These options in turn are defined by **Value of Inaction**:

$$V^i(A,K,p) = \pi(A,K) + \beta E_{A',p'|A,p} V(A', (1-\delta)K, p') \tag{2}$$

- and **Value of Adjustment:**

$$V^a(A,K,p) = \max_{K'} \Big\{ \lambda \cdot \pi(A,K) - F \cdot K - p(K' - (1-\delta)K) + \beta E_{A',p'|A,p} V(A', K', p') \Big\} \tag{3}$$

Featuring two nonconvex costs:

1. adjustment disruption costs $\lambda$, some share of foregone profits
2. fixed costs of adjustment $FK$, which scale with firm size

# 1. Disruption of usual operations

- Two *costs* that are **independent of the level of investment activity.**
- The first is a loss of profit $= (1 - \lambda)\pi(A, K)$
- Opportunity cost of investment, plant must be shut down during a period of investment activity.

- **Byres Road** in 2025: large disruption to install new paving
- **UofG Campus** in 2025: some paths blocked by construction
- Doesn't matter if Keystone Building is 5 floors or 50

## 2. Fixed Costs

**Fixed Costs: "in for a penny, in for a pound"**

- The 2nd nonconvex cost is $F$ (or as a percent of capital lost, $FK$).
- **Fixed Costs** are triggered for any adjustment, independent of size

$$FC_t = \begin{cases} 0 & \textit{if} \quad \mathcal{I}_t = 0 \\ F \times K_t & \textit{if} \quad \mathcal{I}_t \neq 0 \end{cases}$$

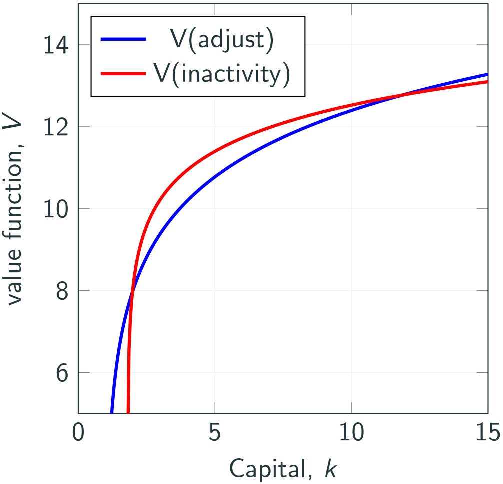
- The inclusion of $K$: while independent of the current level of investment activity, may have some **scale aspects to them**.
- "Getting started (expanding) to any degree is hard"

**Overall** the firm pays $[F \cdot K + (1 - \lambda) \cdot \pi(A, K)]$ to invest, regardless of size of investment!

# Economic Behaviour with Fixed Costs

- **How does the policy function for investment look in this case?**
- Compare relative to the function obtained under a convex cost only?
- The **key feature** of the investment function is that it will entail **jumps**. How?
- Invest **when it is important** (low $K$), **invest in bursts** to avoid paying the fixed cost and disruption more than once.

# Value Functions



# The Economics of Inaction

- **Fixed Costs**: don't vary with size of adjustment, create regions of **inaction**

- Close to the optimal choice, the benefits of adjustment are **small**.

+ Far away from the optimum capital, the **benefits of adjusting are large**.

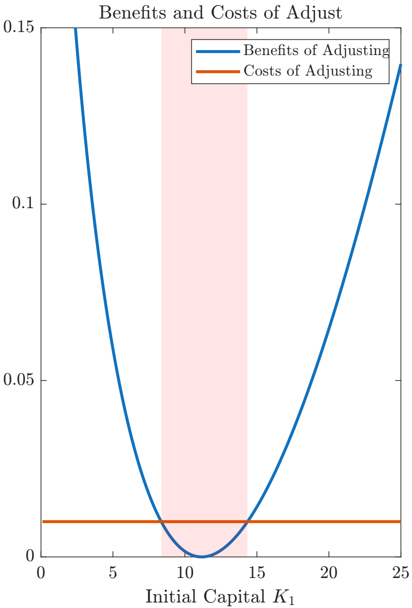
- We can show the **Gains-of-Adjustment (Benefits minus Costs)** curve is **U-shaped** and passes through zero twice:

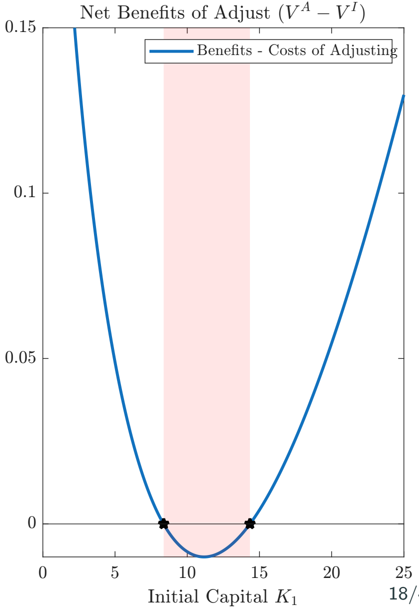
→ Where the desired adjustment is small, it is not worth doing (FC dominates)

→ Where the desired adjustment is large, gains of adjusting are large
  - if $K_t >> K^*$: sell down to $K*$, $I_t < 0$
  - if $K_t << K^*$: invest to $K*$, $I_t > 0$

(!!) This region below zero, is continuous and defines the inaction region ]

# Benefits and Costs of Adjusting with Fixed Costs



# Gains from Adjusting with Fixed Costs

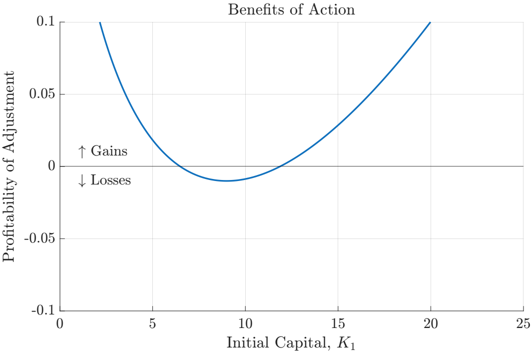



**Figure 3:** Gains from Action: $V^{Adjust} - V^{inaction}$

1

[1] I have used a 2-period model to get clean results. The intuition generalises, but proofs are a nightmare for $T > 2$ for reasons beyond the scope of this course

# Inaction Region

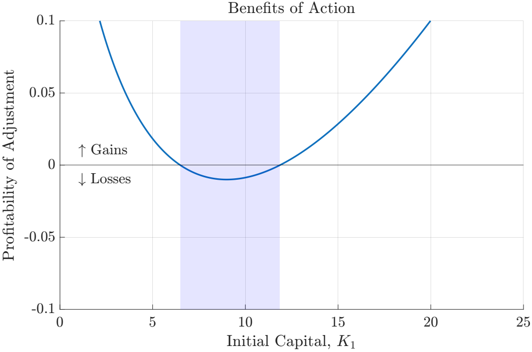
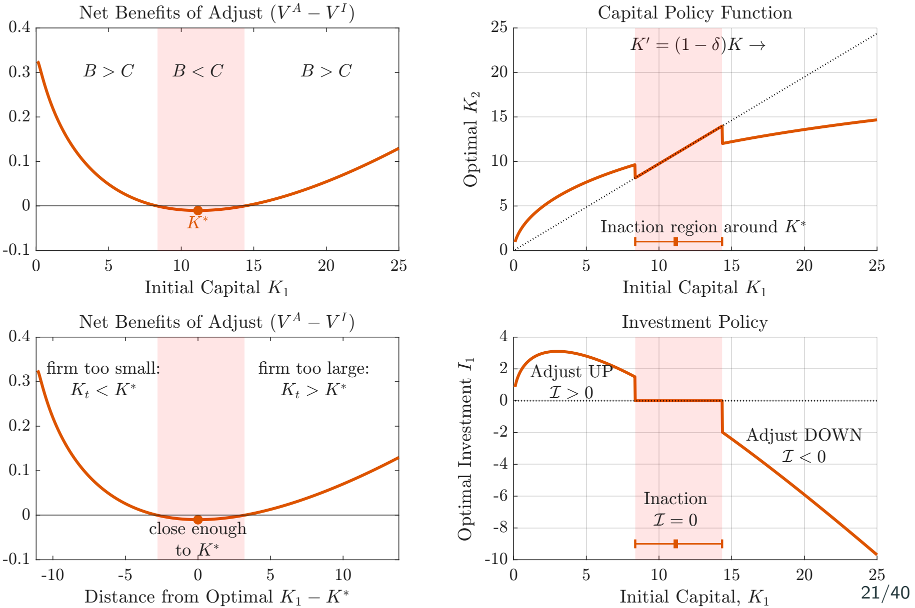


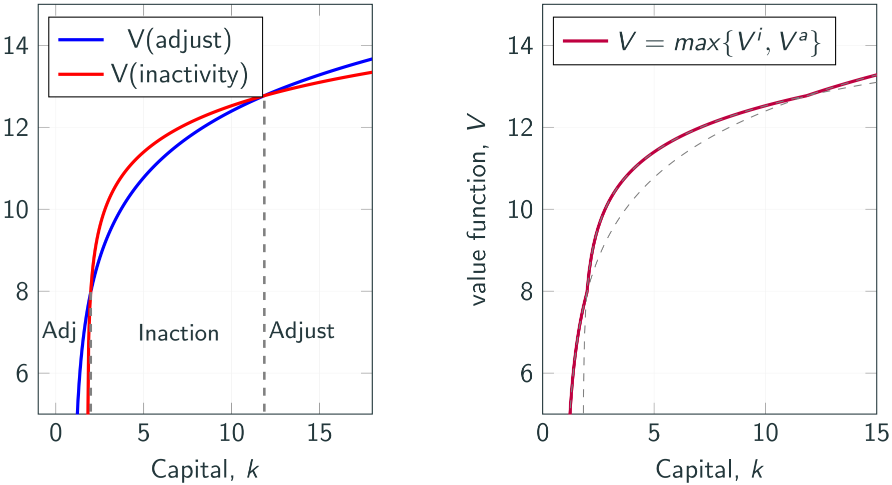

**Figure 4:** Inaction Region highlighted

- Small changes are not worth it: if $K \in [K_L, K_U]$, adjustment creates losses
- **Behavioural Rule**: Adjust when: $V^A \geq V^I$
- equivalent to $K \leq K_L$ or $K \geq K_U$

# Optimal Choices and Regions of Inaction



# The Value of Action and Inaction



# Irreversibility

## British English has a lot of phrases about no going back

- You can't put the toothpaste back in the tube.
- **Can't unburn the toast.**
- Point of no return.
- The cat's out of the bag.
- That ship has sailed.
- Crying over spilt milk.
- Locking the barn door after the horse has bolted.
- That bell can't be unrung.
- You can't unscramble an egg.
- The die is cast.
- Crossing the Rubicon.
- What's done is done.
- I've burned my bridges.
- That's water under the bridge.
- The bird has flown.
- You can't put the genie back in the bottle.
- The bolt is shot.

# Everyone has a lot of idioms about irreversibility

(As far as I can tell) every language has rich idioms to describe irreversibility of life:

- 覆水难收 spilled water cannot be gathered
- 木已成舟 the wood has been made into a boat
- Are there more examples from your language(s)?

**Costly Mistakes:**

- **undoable mistakes and regret** are universal human experiences
- These idioms contain knowledge from generations of **costly mistakes**
- Society can't teach everyone economics
- Culture can give them **memorable phrases that approximate optimal** behaviour under irreversibility.

# Irreversibility as Resale price wedges

## Wedding Dresses and Ferraris

- (e.g.) When I drive my new Ferrari off the dealership, its market value drops considerably ($p_s < 1$)

**Table 2:** Hypothetical **Price Wedge Between New and Resale Value** Across Asset Classes

| Asset | New Price | Resale (unused) | Wedge |
|---|---|---|---|
| Cars | 100% | 70–80% | 20–30% |
| Diamonds | 100% | 30–50% | 50–70% |
| Wedding dress | 100% | 20–40% | 60–80% |
| Books | 100% | 50% | 50% |
| Houses | 100% | 95–98% | 2–5% |
| Land | 100% | $\sim$100% | $\sim$0% |
| Stocks | 100% | $\sim$100% | $\sim$0% |

# Irreversibility as Resale price wedges

A lot of assets / investments have basically **no secondary market for resale**

- **Firm-Specific/Customized Capital**
- Toyota's body-shape moulds and dies cannot be sold to Honda
- Supply chains are highly customised
- Vulnerable to shocks
- Entire Toyota production shut down after 2011 Earthquake. Small affected area, but very industrialised. One supplier of a single compound in paint shut down.
- A degree:
- Time (and money spent) cannot be reversed
- human capital: the degree diploma and skills/credentialls can't be sold to someone else
- (but you can rent your labour and skills)

## So Installations of new capital are often hard to undo

- Extractions or retirements of capital might be hard or impossible (it might destroy the capital)
- Even afterwards, there might be no willing buyer for the used capital

Economists use two terms for these **difficulties**:

- **Irreversibility**: it is hard to undo the installation process
- **Specificity**: the capital is specific to the firm, the capital has low scrap value (no buyer or unusable)

**Examples**:

- Hydroelectic Dam: can't move the river!
- Location-specific Oil pipeline
- Nuclear power plant: Very hard to retire and demolish
- Supply chains are often highly customised to buyers processes (Automobiles).

## So Installations of new capital are often hard to undo

- Extractions or retirements of capital might be hard or impossible (it might destroy the capital)
- Even afterwards, there might be no willing buyer for the used capital

Economists use two terms for these **difficulties**:

- **Irreversibility**: it is hard to undo the installation process
- **Specificity**: the capital is specific to the firm, the capital has low scrap value (no buyer or unusable)

**Examples**:

- Hydroelectic Dam: can't move the river!
- Location-specific Oil pipeline
- Nuclear power plant: Very hard to retire and demolish
- Supply chains are often highly customised to buyers processes (Automobiles).

## So Installations of new capital are often hard to undo

- Extractions or retirements of capital might be hard or impossible (it might destroy the capital)
- Even afterwards, there might be no willing buyer for the used capital

Economists use two terms for these **difficulties**:

- **Irreversibility**: it is hard to undo the installation process
- **Specificity**: the capital is specific to the firm, the capital has low scrap value (no buyer or unusable)

**Examples**:

- Hydroelectic Dam: can't move the river!
- Location-specific Oil pipeline
- Nuclear power plant: Very hard to retire and demolish
- Supply chains are often highly customised to buyers processes (Automobiles).

## Irrevererersibility in the model as a price-gap: $1 \geq p_s$

- We have not distinguished between **buying and selling price** of capital.
- There are several frictions present in the market for used capital that makes them imperfectly suitable for uses at other production sites.
- To allow for this **partial irreversibility**, we alter our optimization problem to d**istinguish the buying and selling prices** of capital.
- Full irreversibility of installtions can be captured as $p_s = 0$

# Irreversibility in the model

- The value function now has three options in the max operator:

$$V(A, K) = \max \left\{ V^b(A, K), V^s(A, K), V^i(A, K) \right\} \tag{4}$$

- for all $(A, K)$: **B**uying / **S**elling / **I**naction

# Irreversibility in the model

- These options care then given by value functions for buying (selling) and choosing investment (retirement) of capital

$$\textit{buying:}\ V^b(A,K) = \max_I \left\{ \pi(A,K) - I + \beta E_{A'|A} V(A', K(1-\delta) + I) \right\} \quad (5)$$

$$\textit{selling:}\ V^s(A,K) = \max_R \left\{ \pi(A,K) + p^s R + \beta E_{A'|A} V(A', K(1-\delta) - R) \right\} \quad (6)$$

- and

$$\textit{inaction:}\ V^i(A,K) = \pi(A,K) + \beta E_{A'|A} V(A', K(1-\delta)) \quad (7)$$

# Irreversibility in the model

- Under the buy option, the plant obtains capital at a cost normalized to one.
- Under the sell option, the plant retires $R$ units of capital at a price $p^s$.
- The third option is inaction so that the capital stock depreciates at a rate of $\delta$.
- Intuitively, the gap between the buying and selling price of capital will produce inaction.
- The sell-buy back round trip is expensive. The costs of inaction might be smaller.

# Inaction towards small negative shocks

- Suppose that there is an adverse shock to the profitability of the plant.
- If this shock was known to be temporary, then selling capital and repurchasing it in the near future.
- ... would not be profitable for the plant as long as $p_s < 1$.[2]
- Thus the manager of the plant does nothing, that is we observe inaction.
- Under the convex cost case, (with $p_s = 1$) an adverse shock to profitability would imply downsizing (selling capital).

- **Car rental companies sold all their cars** in Mar-Apr 2020, then couldn't find any new vehicles to buy in 2021: shortage of rentals, forgone profits! (obviously survival was more important, but highlights Sell-Buy-Back-Later dynamics are usually bad)

[2] Check appendix for derivation: old friend $q$ makes an appearence again

# Real World Applications of Irreversibility

# In Today's Context

- How much of today's capex by large US firms is firm-specific?
- Large AI / datacentre spending – stranded assets?
- Secondary market if market collapses?
- 20th century: Detroit and fall of auto sector
- 20th century: Glasgow: Canals, Railways, Shipbuilding!

# In Today's Context

- How much of today's capex by large US firms is firm-specific?
- Large AI / datacentre spending – stranded assets?
- Secondary market if market collapses?
- 20th century: Detroit and fall of auto sector
- 20th century: Glasgow: Canals, Railways, Shipbuilding!

# In Today's Context

- How much of today's capex by large US firms is firm-specific?
- Large AI / datacentre spending – stranded assets?
- Secondary market if market collapses?
- 20th century: Detroit and fall of auto sector
- 20th century: Glasgow: Canals, Railways, Shipbuilding!

# Specificity & Irreversibility: Glasgow Shipbuilding

**Asset Specificity:**

- Dry docks built for River Clyde
- Giant cranes (no alternative use)
- Specialized workforce

**Complete Irreversibility:**

- Can't relocate infrastructure
- No 2nd market for ship cranes
- Large exit/disassembly costs

# Why Specificity & Irreversibility Matter: Borrowing

**The Glasgow Shipbuilding Death Spiral/Feedback Loop:**

1. Global competition increases → Cash flow drops ($z_L$)
2. Irreversible investments → Can't liquidate assets for cash
3. Specific assets → Can't pledge as collateral for loans ($P_s$ is very low)

$$B < B_{max} = (e.g.)\ 50\% \times P_s(1-\delta)K = 50\% \times \textit{resale value}$$

4. Can't finance modernization → Fall further behind (accelerator!)
5. More cash flow declines → Eventually exit

# Why Specificity & Irreversibility Matter: Borrowing

**The Glasgow Shipbuilding Death Spiral/Feedback Loop:**

1. Global competition increases $\rightarrow$ Cash flow drops ($z_L$)
2. Irreversible investments $\rightarrow$ Can't liquidate assets for cash
3. Specific assets $\rightarrow$ Can't pledge as collateral for loans ($P_s$ is very low)

$$B < B_{max} = (e.g.)\ 50\% \times P_s(1-\delta)K = 50\% \times \textit{resale value}$$

4. Can't finance modernization $\rightarrow$ Fall further behind (accelerator!)
5. More cash flow declines $\rightarrow$ Eventually exit

# Why Specificity & Irreversibility Matter: Borrowing

**The Glasgow Shipbuilding Death Spiral/Feedback Loop:**

1. Global competition increases $\rightarrow$ Cash flow drops ($z_L$)
2. Irreversible investments $\rightarrow$ Can't liquidate assets for cash
3. Specific assets $\rightarrow$ Can't pledge as collateral for loans ($P_s$ is very low)

$$B < B_{max} = (e.g.)\ 50\% \times P_s(1-\delta)K = 50\% \times \textit{resale value}$$

4. Can't finance modernization $\rightarrow$ Fall further behind (accelerator!)
5. More cash flow declines $\rightarrow$ Eventually exit

# Why Specificity & Irreversibility Matter: Borrowing

**The Glasgow Shipbuilding Death Spiral/Feedback Loop:**

1. Global competition increases $\rightarrow$ Cash flow drops ($z_L$)
2. Irreversible investments $\rightarrow$ Can't liquidate assets for cash
3. Specific assets $\rightarrow$ Can't pledge as collateral for loans ($P_s$ is very low)

$$B < B_{max} = (e.g.)\ 50\% \times P_s(1-\delta)K = 50\% \times \textit{resale value}$$

4. Can't finance modernization $\rightarrow$ Fall further behind (accelerator!)
5. More cash flow declines $\rightarrow$ Eventually exit

# Why Specificity & Irreversibility Matter: Borrowing

**The Glasgow Shipbuilding Death Spiral/Feedback Loop:**

1. Global competition increases $\rightarrow$ Cash flow drops ($z_L$)
2. Irreversible investments $\rightarrow$ Can't liquidate assets for cash
3. Specific assets $\rightarrow$ Can't pledge as collateral for loans ($P_s$ is very low)

$$B < B_{max} = (e.g.)\ 50\% \times P_s(1-\delta)K = 50\% \times \textit{resale value}$$

4. Can't finance modernization $\rightarrow$ Fall further behind (accelerator!)
5. More cash flow declines $\rightarrow$ Eventually exit

# Conclusion 1: Fixed Costs

- How important are fixed costs? **Very!**
- We've seen that fixed costs create an area of small adjustments which, if done, would lead to losses
- in the case of potential losses, the decision is simple: **inaction**!
- **Economic Behaviour**: Fixed Costs create a range of inaction, with bounds $[U, L]$
- many aspects of economics, finance, life(!) feature fixed costs of switching from one mode to another
  - relocation to new city (constant changing opportunity, move infrequently)
  - retraining
  - mental context switching
  - cooking one meal versus batching
  - only check emails a few times a day (constant arrivals)

# Conclusion 2: Irreversibility and Specificity

- How important is investment irreversibility? **Very!**
- When capital goods are **highly specialized** on industry specific, firms may find that **reversing an investment** decision is impossssible or costly
- **The Ferrari / Wedding Dress effect**: difference between the purchase price and resale price of capital.
- Irreversibility may generate a **"reluctance to invest"**.
- **Low Borrowing Capacity:** can reduce collateral value of assets (borrowing constrained)
- **Can harm future flexibility** to respond to changing business conditions. Flexibility is also an asset (next week!)

# Appendix

# Optional Appendix: Partial Irreversibility and Inaction

We would want to sell when the value of selling is higher than of inaction:

$$V^i = \pi(A_{low}, K) + \beta E_{A'|A} V(A', K(1-\delta)) \tag{8}$$

$$V^s = \pi(A_{low}, K) + 0.6R + \beta E_{A'|A} V(A', K(1-\delta) - R) \tag{9}$$

Let's say profits in both scenarios are the same today $\pi(A_{low}, K)$, what matters is continuation values. We prefer to keep the capital and not to sell when $V^i > V^s$, for some retirement of capital $R$

$$\beta E_{A'|A}[V(A', K(1-\delta)) - V(A', K(1-\delta) - R)] > 0.6R \tag{10}$$

We produce and then decide whether to sell or not.

As R gets very small, we can substitute with marginal value $V_K$, recall the meaning of this term from the Tobin unit.

$$q^{IRR} = \beta E_{A'|A}[V_K(A', K(1-\delta))] > p_s = 0.6 \tag{11}$$

$$\textbf{profits over useful lifetime} \text{ of capital} > \text{revenue raised today by selling} \tag{12}$$

$$\textbf{Shadow Price} > \text{Market Price for resold capital} \tag{13}$$

This rule says keep your capital if the expected continuation value is sufficiently high compared to sale value. This depends on the chances conditions $A'|A$ improve, how durable is the asset $\delta$, market prices for used assets etc etc.

$$q \geq p_s$$

So the **inaction set** is pinned down by the range

$$:= InactionSet, \mathbb{I} : \{(A, K) : 1 > q(A, K) > p_s\}$$