Testing Q theory

Investment, Finance & Asset Prices ECON5068

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Lecture Overview

- Marginal Q vs Average Q
- Empirical Implementation of Q Model
- Results and Issues
- Further Reading:
    1. Hayashi (1982), " Tobin's Marginal Q and Average Q: A Neoclassical Interpretation ", Econometrica.

Investment Q Model

- This model predicts that **investment and Q** are related.
- **Knowing Q** is sufficient to predict what the investment level will be.
- Is Q readily observable in the data?

Investment Q Model

- Remember Q is given by,

$$Q = \beta \mathbb{E}[V_{K'}(\theta', K')] \tag{1}$$

- the **derivative** of the **value function**, hence called **Marginal** Q.
- To test Q theory, we need to **measure this value**.
- The marginal value is unobservable making empirical testing of investment Q models **extremely difficult**.

Investment Q Model

- As the value function and hence its derivative is not observable, **testing adjustment cost investment model is difficult**.
- We need to find a suitable **proxy** for the derivative of the value function.
- A good proxy is to substitute the marginal value of the firm with its average value.
- **Under what conditions** is the average value a good proxy for the marginal value?

Hayashi's Result

- **Hayashi (1982)** established some conditions under which the average and marginal Q coincide.

Proposition

Hayashi showed that a firm's marginal Q equals average Q when the following three conditions are satisfied:

1. The firm is a **price taker** in the output market.
2. Production **function is linearly homogenous**.
3. Cost functions are **linearly homogenous**.

Linearly Homogeneous Functions

Definition

A function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}$ is homogeneous of degree r if

$$f(tx_1, tx_2 \ldots tx_n) = t^r f(x_1, x_2, \ldots x_n) \tag{2}$$

for all $r > 0$ where $x = (x_1, x_2, \ldots x_n)$.

- Homogeneous functions of degree one $(r = 1)$ are called linearly homogeneous.
- The **Cobb Douglas** production function of the form $f(k, l) = k^\alpha l^{1-\alpha}$ is an example of a **linearly homogeneous function**.

Linearly Homogeneous Functions

Two properties of degree r differentiable homogeneous functions:

1. Each first order partial derivative $\frac{\partial f}{\partial x_i}$ is a homogeneneous function of degree $r - 1$.

2. Euler's Theorem: $\sum_{i=1}^{n} x_i \left(\frac{\partial f(x)}{\partial x_i} \right) = r \cdot f(x)$. The Euler's theorem is a sufficient condition to prove the homogeneity of the function.

Hayashi's Result

- The first condition says that the firm is operating in a **perfectly competitive** environment such that it has **no power in the pricing** of its output product. The market determines the product price.

- **Linear** homogeneity means homogenous functions of **order one**. (Satisfies scaling property).

- A production function which is homogenous of degree one displays constant returns to scale (CRS).

Hayashi's Result

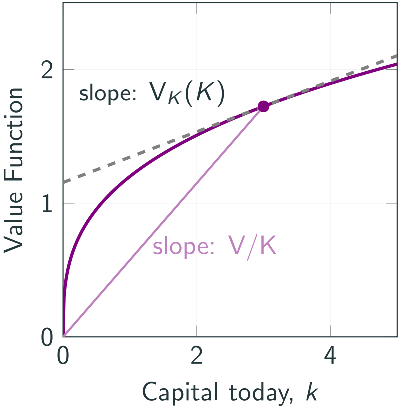
- Hayashi's results form the basis of empirically implementing the investment Q model and facilitates its testing.

- We can now substitute the unobservable marginal Q with the observable average Q.

- That is,

$$\boxed{\text{Marginal Q} = \mathbb{E}[V_{K'}(\theta', K')] = \frac{V}{K} = \text{Average Q}} \tag{3}$$

Value, Marginal Q and Average Q



- Hayashi says we can only assume marginal Q ≈ Average Q if the Value function is "well behaved"

Empirical Testing

- Tests of Q theory on **panel data** are frequently conducted using the following empirical specification:
- The **data** two dimensions: N-firms, T-years, NT-observations

$$\boxed{\frac{I_{it}}{K_{it-1}} = a_i + a_1 Q_{it} + a_2 X_{it} + \varepsilon_{it}} \tag{4}$$

- This is a **panel regression**, where the subscript (i) denotes the individual firm (or plant) and (t) denotes time.
- We are regressing investment rate (I/K) on Q and control(s) X, ε is an error term.
- **Regression**: fitting the (a_i, a_1, a_2)-parameters that best fit the data, the estimates numbers are usually represented by hats, $\hat{a}$

Empirical Testing

- What is the role of these controls X_{it}? ...
- **Hold on!** Q theory says that the value of Q is sufficient to incorporate all relevant information related to production and investment.
- That is, **no other variable should impact investment**.
- These variables X_{it} are included as a means of testing the theory, where the theory predicts that these variables from the information set should be insignificant.
- In other words, the coefficient a_2 should be statistically insignificant (meaning, a statistical zero)

Empirical Testing

- The estimate of the coefficient $\hat{a}_1$ also provides information on the adjustment cost parameter.
- This comes from a modification of the adjustment costs to the form:

$$C(K', K) = \frac{\phi}{2}\left(\frac{K' - (1 - \delta K)}{K} - a_i\right)^2 \tag{5}$$

$$\frac{I}{K} \approx \frac{1}{\phi}(Q - 1) + a_i \tag{6}$$

$$\texttt{regression:} \Rightarrow (I/K)_{it} = constant + a_1 Q + a_i + \varepsilon_{it} \tag{7}$$

Lecture Overview

- Empirical Implementation of Q Model - Fazzaari, Hubbard, Petersen (1988)
- Reading:
  1. Steven M. Fazzari, R. Glenn Hubbard, Bruce C. Petersen, "Financing Constraints and Corporate Investment", Brookings Papers on Economic Activity.

Empirical Testing

- We will focus on three seminal papers in the investment literature -
  - **Fazzari, Hubbard, Petersen (1988)**,
  - Gilchrist and Himmelberg (1995)
  - Cooper and Ejarque (2003).

- These papers make use of Hayashi's proposition and test the Q theory of investment.

- Question: Can investment be entirely explained by average Q?
  1. If yes, then this validates the Q model of investment.
  2. If no, what other variables are important in explaining firm level investment?

FHP 1988

- We will start with the paper by Fazzari, Hubbard, Petersen (1988).
- FHP postulate a regression of the form:

$$\frac{I_{it}}{K_{it-1}} = a_0 + a_1 Q_{it} + a_2 (\frac{CF_{it}}{K_{it-1}}) + \epsilon_{it} \tag{8}$$

- where the investment rate is regressed on average Qand cashflow (scaled by capital).
- The idea is that if there are financial constraints (or capital market imperfections), then these will be felt more at firms that are either unable to find it or difficult to obtain at a reasonable cost.
- Think of a small firm, few assets to pledge as collateral, no credit record

Diversion: what are financial constraints?

- So far in Tobin's model we haven't talked at all about **borrowing**
- We assume the firm calculates optimal investment, and acts
- **FINANCE!** We have hidden a major component of firm dynamics!

Imagine we have very low k_t today, but expected profits are very high so q is high, and desired investment is large

$$dividend = \theta k^{\alpha} - p\mathcal{I} - \mathcal{AC}(\mathcal{I}, k)$$

We haven't restricted dividends in anyway, they could be negative

- To make up the gap between expenditures and income, the firm must borrow!

Diversion: what are financial constraints?

- Gap between expenditures and income, the **firm must borrow**!
- **External finance** (banks, markets, lenders) is **costly**
- default/monitoring costs, asymmetric information, lender profit margins

Assume the firm pays a **financing cost** of λ-percent on its external borrowing

$$\begin{aligned} \mathcal{FC}(\mathcal{I}, k) &= \lambda^{EXT} \cdot \max(0, p\mathcal{I} - \theta k^{\alpha}) \\ &= \lambda^{EXT} \cdot Borrowing \end{aligned} \tag{9}$$

- We would expect **premium to be higher** for small firms: $\lambda^{EXT}(k)$
- (**Credit limits** on $B \leq B_{max}$ are probably tighter too)
- Hurts small firms trying to expand

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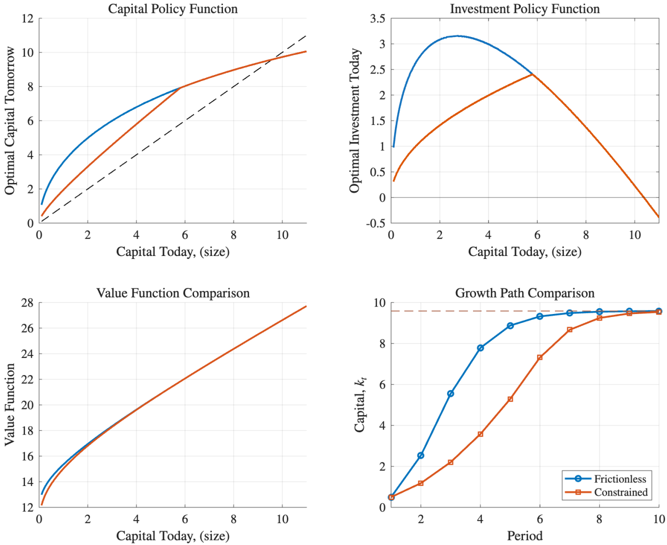

Figure 1: Tobin model (blue) v Tobin + Financial Constraints (orange)

FHP 1988

- If their idea is true, then investment for financially constrained firms will depend on variables **other than Q** - compare the blue and orange lines in the model.

- Testing the coefficient a_2 might provide evidence for this conjecture: does internal finance meaningfully comove with investment?

- The control variable suggested by FHP is **cashflow**.

FHP 1988: Earnings paid as dividends or kept as cash

They classify firms into three groups:

1. Class 1: Very Low Payout Rate: Dividends/Income < 0.1
2. Class 2: Low Payout Rate: $0.1 <$ Dividends/Income < 0.2
3. Class 3: All other firms.

- Who wants to save their operating profits, who is paying dividends?
- Of these, how much investment is internally funded through profits

FHP 1988 - Data and Q Measurement

Data:

1. Value Line, US firms
2. Time period: 1970-1984
3. Manufacturing Sector

Measuring Q:

1. Q at the beginning of the period (so really Q_{it-1}, data is EOP accoutning)
2. Ratio of market value (value of equity plus value of debt) over replacement value of capital.

Summary Statistics: Manufacturing Firms, 1970–84

Statistic	Class 1[a]	Class 2[b]	Class 3[c]
Number of firms	49	39	334
Average retention ratio (1-payout rate)	0.94	0.83	0.58
Percent of years with positive dividends	33	83	98
Average real sales growth (% per year)	13.7	8.7	4.6
Average investment–capital (I/K) ratio	0.26	0.18	0.12
Average cash flow–capital (CF/K) ratio	0.30	0.26	0.21
Average correlation of cash flow with investment[d]	0.92	0.82	0.20
Average firm SD of investment–capital ratios	0.17	0.09	0.06
Average firm SD of cash flow–capital ratios	0.20	0.09	0.06
Average capital stock, 1970 (millions of 1982 $)	100.6	289.7	1,270.0
Median capital stock, 1970	27.1	54.2	401.6
Average capital stock, 1984	320.0	653.4	2,190.6
Median capital stock, 1984	94.9	192.5	480.8

Notes: Firms classified by dividend–income ratios. [a] Dividend–income ratio < 0.1. [b] $0.1 \leq$ ratio < 0.2. [c] Ratio ≥ 0.2. [d] Based on time series within each firm.

FHP 1988 - Sample Selection

The **summary table** suggests the following:

1. C1 and C2 firms have **retained most income** 83,94% v 58%
2. Are relatively **small** (C3 x10-x13 larger)
3. Have experience **most growth in their capital** stock.(x3 v x1.2)
4. Exhaust almost all of their **cash flow in their investment** (87 percent).
5. They also exhibit a much **higher correlation of investment to cash flow**. 0.92,0.82 v 0.20

C1 and C2 act in ways suggesting they are constrained!

	Class 1	Class 2	Class 3
Early Years: *1970–75*			
Q_{it}	-0.0010***	0.0072**	0.0014***
	(0.0004)	(0.0017)	(0.0004)
$(CF/K)_{it}$	0.670***	0.349***	0.254***
	(0.044)	(0.075)	(0.022)
R^2	0.55	0.19	0.13
Medium Duration: *1970–79*			
Q_{it}	0.0002	0.0060**	0.0020***
	(0.0004)	(0.0011)	(0.0003)
$(CF/K)_{it}$	0.540***	0.313***	0.185***
	(0.036)	(0.054)	(0.013)
R^2	0.47	0.20	0.14
Full Sample: 1970–84			
Q_{it}	0.0008*	0.0046***	0.0020***
	(0.0004)	(0.0009)	(0.0003)
$(CF/K)_{it}$	0.461***	0.363***	0.230***
	(0.039)	(0.039)	(0.010)
R^2	0.46	0.28	0.19

(the number in brackets tells us the uncertainty of the estimate, if the ratio of the two is greater than 1.96, effect is unlikely to come from a true zero plus statistical noise)

FHP 1988 - Results

- More **financially constrained** firms (Class 1) exhibit significantly greater **investment-cashflow sensitivities** than firms that appear less financially constrained.

- **Difference** in cashflow coefficients between different classes of firms are always statistically significant.

- **Q is a bit of a mess!** coefficients vary by **significance** (insignificant is not statistically different from zero) and by **sign**

- R^2 is always higher for these firms. (this measures how much variance in I/K can be explained by variation in the RHS variables, NOT a measure of causality however)

FHP 1988 - Robustness

These results are robust to a number of checks:

1. Possible measurement errors in Q.
2. Using lagged values (last period for instance) of Q and CF/K.
3. Using end of period Q.
4. Alternate measurement of Q by using sales.
5. Results are consistent across different industries.

FHP 1988 - Issues

Possible issues with their approach:

1. Q may not be accurately reflecting **market fundamentals** due to excessive volatility in equity market.

2. **Measurement errors** in Q could be correlated with cash flow.

3. **Cash flow may reflect news** about future profitability of investment that is not captured by beginning of period Q.

Cooper-Ejarque (2003) in one slide...

- Model can generate positive correlation between (I/K) and CF even in a model with **no financial constraints**
- **No wedge** between internal and external finance needed
- Strictly **concave profit** function splits AvgQ and marginalQ

Imagine shocks to z_{it}, probably not observed directly:

$$InvestmentRate(z)_{it} = a_0 + a_1 Q_{it} + a_2 Cashflow(z)_{it} + \epsilon_{it} \tag{10}$$

This breaks the regression $Cov(CF, \epsilon) > 0$, the estimated a_2 is **biased upwards!**

… from my own work

- cash ↔ investment is **dynamic**: build cash before investment/hiring episode, run down liquidity during and after (top row)
- Precaution: firms anticipate future constraints and save strategically (larger group of firms than just the directly constrained)

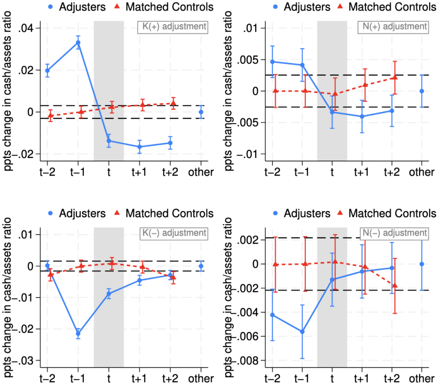

Figure 2: Cash dynamics around expansions/contractions in K and N