Investment

Investment, Finance and Asset Prices ECON5068

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Lecture Overview

- **What is economic investment?**
- Some **micro** (firm-level) and **macro** (economy-level) empirical **facts** about investment
- How do firms **choose investment**, User Cost Model

Definitions

- **Investment**: accumulation of the means of production
- **Capital:** *(typically physical), long-lived assets* that can be combined with labour and materials in *production* to create output
- In **National Accounts** physical investment is:
  - called **Gross Fixed Capital Formation**
  - residential and nonresidential buildings
  - equipment, machines, tools, computers
  - cars, furniture
- In **firm accounts**, called **PPE** (property, plant, equipment)

Investment Facts

- **BUT** modern economies undertake very large investments **beyond physical machinery/physical assets**
  - research and development
  - employee training
  - brand recognition, customer base and loyalty
  - databases, code, software
  - ⇒ Intangibles meet *economic criteria* (durable, productive) harder for accountants to *measure and value*
- Investment can also include housing or residential assets.
- Broadly, investment = cost today + uncertain future benefit.

➤ we will focus on **tangible investment in PPE**, but in reality capex is some **mix of all of these** (tech company data centres for example).

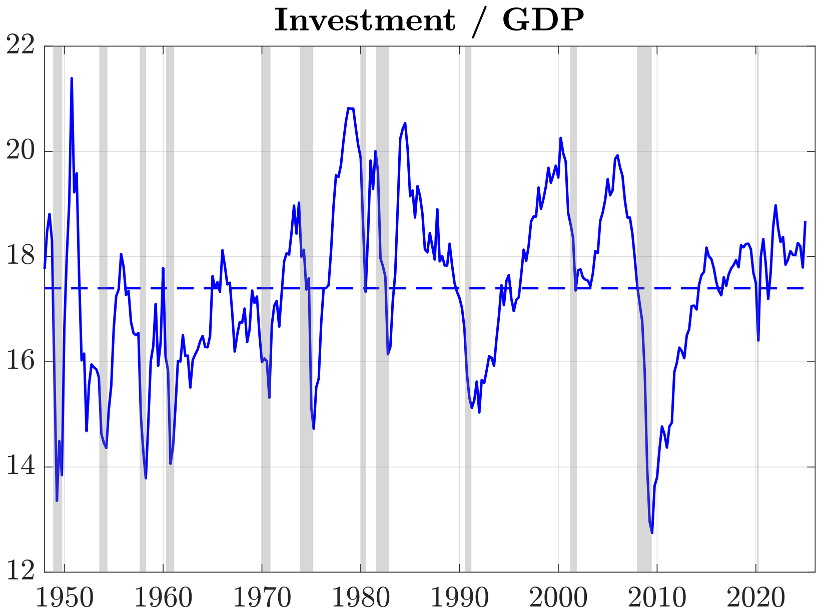
Investment Facts

- **Size of investment varies by country: 10–20% of GDP in developed economies.**

- Investment is highly volatile: $\sim$ 3x–4x more volatile than GDP.

- Small changes in GDP $\Rightarrow$ big changes in investment.

United States Investment-GDP %



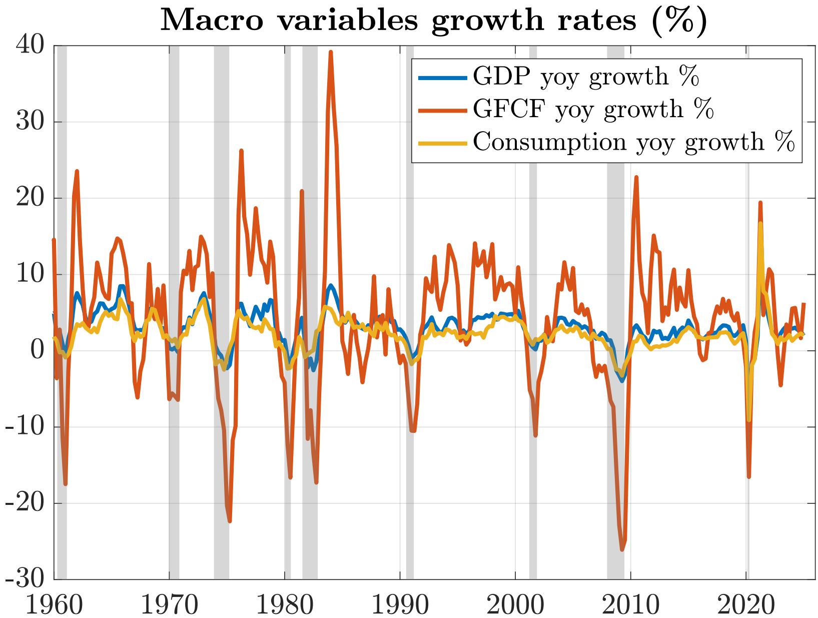
Investment Facts

- Size of investment varies by country: 12–20% of GDP in developed economies.

- **Investment is highly volatile: $\sim$ 3x–4x more volatile than GDP.**

- Small changes in GDP $\Rightarrow$ big changes in investment.

Very Sensitive to Business Cycle Conditions



σ(GDP gr)	σ(Cons gr)	σ(Invest gr)
2.64	2.26	11.29

Table 1: Variation in macro variables

- Some consumption smoothing, std dev cons growth is less than GDP gr
- Investment is amplified, reacts very strongly to the business cycle

Investment - From Aggregate to the Firm Level

- Aggregate statistics give macro perspective.
- To properly understand the behavior and determinants of aggregate investment we need to study investment at the **firm/business level**.

Investment - Firm Level (Apple)

Apple Inc.

CONSOLIDATED BALANCE SHEETS
(In millions, except number of shares, which are reflected in thousands, and par value)

	September 28, 2024	September 30, 2023
ASSETS:		
Current assets:		
Cash and cash equivalents	$ 29,943	$ 29,965
Marketable securities	35,228	31,590
Accounts receivable, net	33,410	29,508
Vendor non-trade receivables	32,833	31,477
Inventories	7,286	6,331
Other current assets	14,287	14,695
Total current assets	152,987	143,566
Non-current assets:		
Marketable securities	91,479	100,544
Property, plant and equipment, net	45,680	43,715
Other non-current assets	74,834	64,758
Total non-current assets	211,993	209,017
Total assets	$ 364,980	$ 352,583

- Capital = Property, Plant, Equipment
- Investment = changes in PPE (plus depreciation)

Investment - Firm Level (BP)

Financial summary

$bn	FY23	FY24
Brent ($/bbl)	82.6	80.8
Henry Hub ($/mmbtu)	2.7	2.3
NBP (p/therm)	98.9	83.6
RMM ($/bbl)	25.8	17.7
Underlying RCPBIT*	27.0	20.6
Gas and low carbon energy	8.7	6.8
Oil production & operations	12.8	11.9
Customer & products	6.4	2.5
Other businesses and corporate	(0.9)	(0.6)
Consolidation adjustment – UPII*	(0.0)	(0.0)
Finance costs	(3.2)	(4.0)
Tax	(9.4)	(6.9)
Non-controlling interests	(0.6)	(0.8)
Underlying replacement cost profit*	13.8	8.9
IFRS operating cash flow*	32.0	27.3
Working capital release*[1]	2.8	0.8
Capital expenditure*	(16.3)	(16.2)
Divestment and other proceeds	1.8	4.2
Share buyback executed[2]	(7.9)	(7.1)
Net debt*	20.9	23.0

(1) Adjusted for inventory holding gains or losses*, fair value accounting effects* and other adjusting items
(2) Includes share buybacks to offset the expected full-year dilution from the vesting of awards under employee share schemes. In 2023, bp completed $675m and in 2024, bp completed the $0.3bn buyback programme

4Q and FY 2024 results

Figure 3: BP Capex

- BP reports capex as investment: 16.2 bn USD (16.2 percent of 100.24 bn PPE 2023)

Firm-level Investment Rates

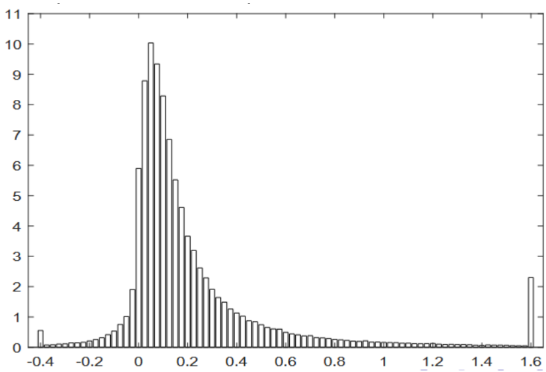


Figure 4: Distribution of Plant-level Investment Rates (% lagged capital)

- Asymmetric dist. of changes in capital stocks
- Negative, Inactive (≈ 0) and Positive Investment Rates.
- Negative and Positive Spikes (v large changes).

How many firms in an economy? (...also, what's a firm?)

- US Small Bus. Assoc.: 34.75M (82 % have no employees)[1]
- US Census Bureau: 6.4M (79 %: $<$ 10 employees) [2].
- UK HOC Library: 5.4M (74 %: 0 employees)
- China: 58.3M private enterprises / own legal entity 32.8M

Takeaways:

- Many small firms with few or no employees (excl. owner)
- The top X% of firms matter for aggregates

[1] https://advocacy.sba.gov/wp-content/uploads/2024/12/Frequently-Asked-Questions-About-Small-Business$_2024 - 508.pdf?utm_source = chatgpt.com$

[2] https://www.census.gov/data/tables/2022/econ/susb/2022-susb-annual.html

Number of Firms

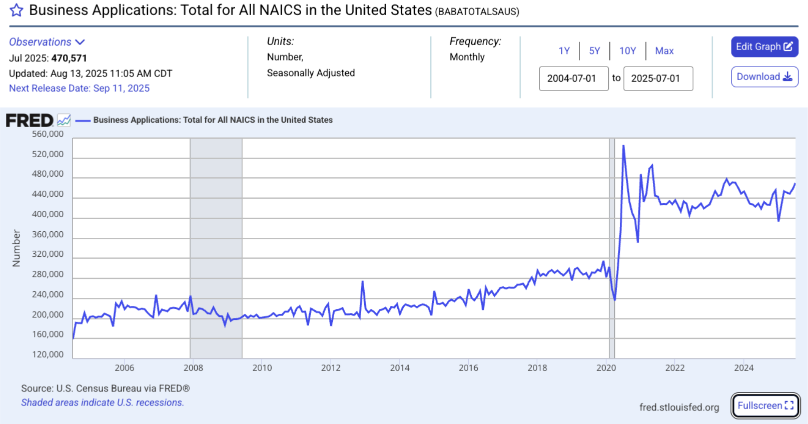
Table 1.
Comparison of Annual Growth of U.S. Nonemployer and Employer Establishments: 2012–2023

Year	Nonemployer establishments	Employer establishments	Nonemployer annual percent change	Employer annual percent change
2012	22,735,915	7,431,808	X	X
2013	23,005,620	7,488,353	1.2	0.8
2014	23,836,937	7,563,084	3.6	1.0
2015	24,331,403	7,663,938	2.1	1.3
2016	24,813,048	7,757,807	2.0	1.2
2017	25,701,671	7,860,674	3.6	1.3
2018	26,485,532	7,912,405	3.0	0.7
2019	27,104,006	7,959,103	2.3	0.6
2020	27,151,987	8,000,178	0.2	0.5
2021	28,477,518	8,148,606	4.9	1.9
2022	28,811,495	8,298,562	4.7	1.8
2023	30,427,808	8,361,342	2.1	0.8

Note: X Not included in this analysis.
Source: U.S. Census Bureau, 2012–2023 Nonemployer Statistics and 2012–2023 County Business Patterns.

Figure 5: Many Sole Traders, not many Employer-Businesses

Entry of New Entrepreneurs/ firms of the future?



- Explosion in new business formation after 2020 in US
- Similar growth in UK, mostly online retail

Entry and Exit from the Stock Market

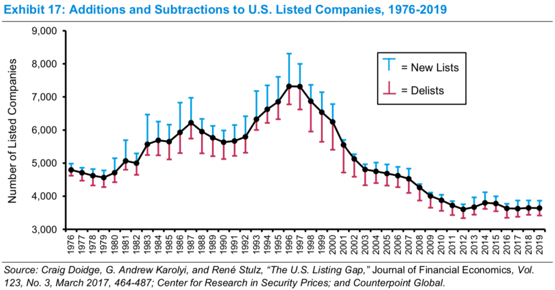


Figure 6: Listing and Delisting Rates on Stock Markets

- Number of public firms is declining.
- High entry and exit rates.

First Model of the Firm's Investment: The User Cost Model

Firm Level Investment Decision

- Let's focus on: **How do firms decide how much to invest?**
- Firms face benefits and costs when investing.
- **Benefits**: higher capital $\Rightarrow$ more output and profit.
- **Costs**: buying and using capital.
- Optimal investment: **marginal benefit = marginal cost**.
- Let's start with a simple but powerful two-period model of investment to build intuition

2 Period Investment Model - Setup

This simple set up helps build intuition and keeps mechanisms clear:

- Firm lives for $\mathbf{T} = \mathbf{2}$ **periods**, starts with K_1 t**aken as given**.
- **Invests** $\mathcal{I}_1$, with **cost** $= P_1^K \times \mathcal{I}_1$.
- **units** of fresh capital installed: $\mathcal{I}_1$, **price** per unit: P_1^K
- Capital (in quantities) follow **law of motion (LOM)**:

$$K_2 = (1 - \delta)K_1 + \mathcal{I}_1 \tag{1}$$

2 Period Investment Model - Timeline

Period 1:

1. Start with K_1 (taken as given)
2. Choose today's L_1 , invest $\mathcal{I}_1$ (pins down K_2) to max profits
3. Distribute dividends D_1

Period 2:

1. Start with K_2
2. Choose L_2 to produce with K_2
3. liquidate firm
4. Distribute D_2 with final operating profit and scrap value

2 Period Investment Model - Remarks

- Activities follow yearly sequence.
- Investment has a one-period delay. This assumption is called **Time-to-build**
- It means if I install new capital today, it is only **productive next period**.
- Value of firm maximized at $t = 0$; future discounted to present value.

2 Period Investment Model - Present Value

- An amount of money (resources generally) today is **worth more than the same amount in the future.**
- **Why?** Interest. \$1 today is worth $\$(1+r)$ one period from now and $\$(1+r)^2$ two periods from now, where r is the interest rate.
- **Present Value** is the current value of a future sum of money or stream of cashflows given some return.
- $\text{Present Value}(t) = \dfrac{\text{Future Value}(t+n)}{(1+r)^n}$
- Discount rate: r, periods: n
- Used for discounting dividends D_2

2 Period Investment Model - Profits

Rewrite operating profit as [revenue - cost of goods sold (COGS)]

$$\pi(K_t) = \max_{L_t} \left\{ F(K_t, L_t) - wL_t \right\}, \quad t = 1, 2 \tag{2}$$

- Production function $F(K, L)$ is concave.
- Output price normalized to 1 (Ouptut = Revenue).
- Competitive firm takes w as given

2 Period Investment Model - Production Function

- A production function is a technology that transforms inputs into output.

$$Output = f(Input_1, Input_2 ... Input_n) = f(\mathbf{x}_n)$$

- An example is the Cobb-Douglas production function:

$$F(K, L) = K^{\alpha} L^{1-\alpha}$$

where α is a parameter that usually takes the value $1/3$.

- First derivative with respect to inputs is called the marginal product:

$$\text{Marginal product of capital} = \text{MPK} = \frac{\partial F(K, L)}{\partial K}$$

$$\text{Marginal product of labour} = \text{MPL} = \frac{\partial F(K, L)}{\partial L}$$

2 Period Investment Model - Operating Profits

- Operating profits is the maximum value given by the solution to eq. (2)
- FOC: $\partial F/\partial L - w = 0$
- Optimal labour chosen s.t. MPL equals wage rate
- The value of profits at L_t^* gives us the maximum value of profits, $\pi(K_t)$ for both periods t = 1,2.

2 Period Investment Model - Dividends

All profits left after costs are distributed as dividends to the shareholders. Dividend at each period is given by the operating profits net of investment costs. Dividends for period 1 is:

$$D_1 = \pi(K_1) - \mathcal{I}_1 P_1^K \tag{3}$$

and for period 2 is given by:

$$D_2 = \pi(K_2) + (1 - \delta) P_2^K K_2 \tag{4}$$

firm sells off any left over capital at the end of period-2

$$K_3 = 0 \Rightarrow I_2 = -(1 - \delta) K_2 \tag{5}$$

2 Period Investment Model - Optimization Problem

The main objective of the firm is to maximize the shareholder wealth or in other words, the present discounted sum of dividends:

$$\max_{\mathcal{I}_1} \left\{ D_1 + \frac{1}{1+r} D_2 \right\} \tag{6}$$

$$\text{s.t.:} \quad K_2 = (1-\delta)K_1 + \mathcal{I}_1 \tag{7}$$

The discounting is done using the real interest rate, r . We need to discount as dividends received in the future is worth less than current dividends. This can be rewritten as:

$$\max_{\mathcal{I}_1} \left\{ \pi(K_1) - \mathcal{I}_1 P_1^K + \frac{1}{1+r} \left(\pi(K_2) + (1-\delta) P_2^K K_2 \right) \right. \tag{8}$$

$$\text{s.t.:} \quad K_2 = (1-\delta)K_1 + \mathcal{I}_1$$

2 Period Investment Model - Substitute and take FOC

Substitute for K_2 to get (8) in terms of $K_1, \mathcal{I}_1$ and δ

$$\max_{\mathcal{I}_1} \Bigg\{ \pi(K_1) - \mathcal{I}_1 P_1^K + \frac{1}{1+r}\Big(\pi((1-\delta)K_1 + \mathcal{I}_1) + (1-\delta)P_2^K((1-\delta)K_1 + \mathcal{I}_1)\Big)\Bigg\} \tag{9}$$

2 Period Investment Model - FOC

Substitute constraint and differentiate wrt $\mathcal{I}_1$:

$$-P_1^K + \frac{1}{1+r}\left(\frac{\partial \pi(K_2)}{\partial K_2} \cdot 1 + (1-\delta)P_2^K\right) = 0$$

- Note that $\pi(K_2(.))$ is a composite function where $\pi(.)$ is the outer function and $K_2(\mathcal{I}_1)$ is the inner function, so we have to use the chain rule.

$$\frac{\partial \pi(K_2)}{\partial \mathcal{I}_1} = \frac{\partial \pi(K_2)}{\partial K_2} \cdot \frac{\partial K_2}{\partial \mathcal{I}_1} = \frac{\partial \pi(K_2)}{\partial K_2}$$

- Let $\pi_K(K_2) = \partial \pi / \partial K_2$, which we will call MPK from now on, not F_K

2 Period Investment Model - Solution

$$P_1^K = \frac{1}{1+r}\left(\pi_K(K_2) + (1-\delta)P_2^K\right)$$

Or, with some rearranging:

$$\pi_K(K_2) = (1+r)P_1^K - (1-\delta)P_2^K$$

Solution

A firm should invest in capital until the value of the extra output that capital produces falls to equal the user cost

$$\pi_K(K_2) = (1+r)P_1^K - (1-\delta)P_2^K$$

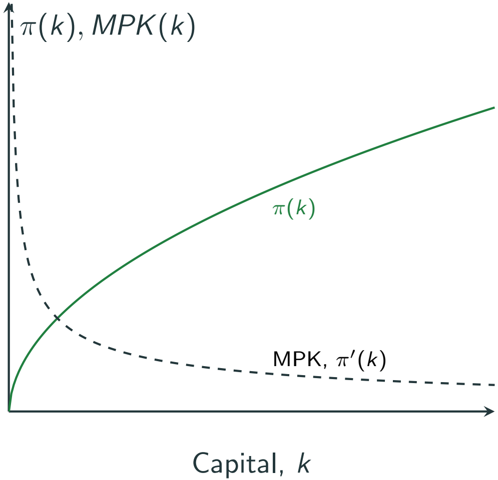
$$\text{Marginal Product of Capital} = \textit{User Cost of Capital}$$

2 Period Investment Model - Investment Decision

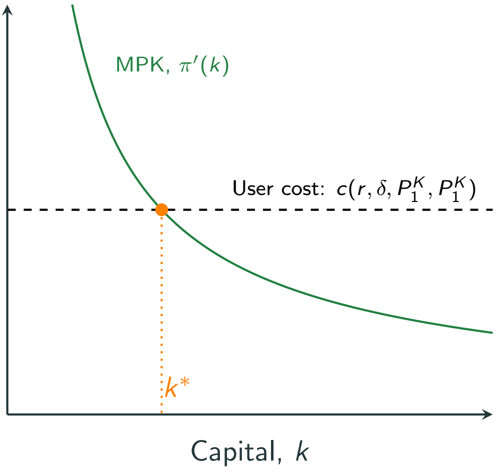
$$\text{MPK} = \text{User Cost of Capital}$$

- **MPK decreasing** in K_2 when $\pi(k)$ is concave.
- User Cost depends on r, δ, P_1^K, P_2^K but not capital
  - outside returns r
  - speed of depreciation δ
  - repricing effects $P_2 - P_1$
  - UC doesn't depend on K so constant function

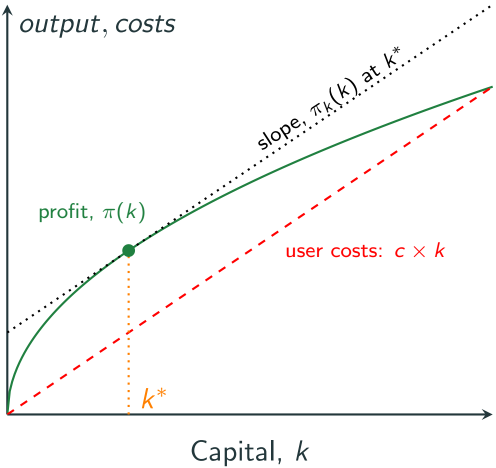
Production function and marginal product



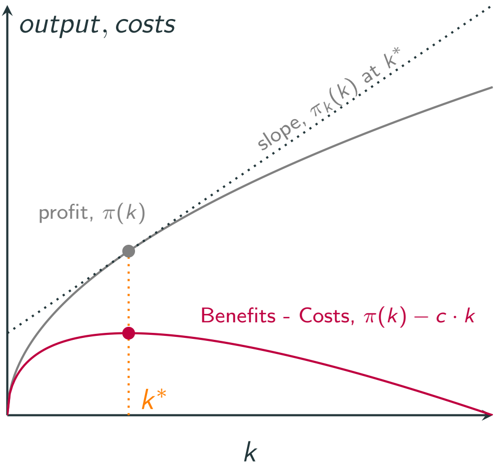
Marginal Profit and User Cost



Back to levels not marginals



Maximising Net Benefits



2 Period Investment Model - Interpretation

- Special case: If the price of capital is constant, $P_1^K = P_2^K$, then

$$\pi_K(K_2) = (r + \delta)P^K \tag{10}$$

- **The benefit** is unchanged, the marginal profits from slightly more capital ($\pi_K(K_2)$)
- **The cost** of using one unit of capital is the loss of **interest foregone** by buying a machine instead of saving the money (rP^K), plus the **depreciation** (maintenance) cost of capital (δP^K).

Investment Decision - Arbitrage Reasoning

Can also use simple **asset pricing logic**:

- Compare **capital investment vs. saving in bank.**
- **No Arbitrage** condition: equalise returns (assumptions?).
- If one or the other had higher return, then the investor would not be maximising profits
- can always allocate more investment towards the high return activity and get more net profits
- $rP_1^K = MPK + (P_2^K - P_1^K) - \delta P_2^K$

Investment Decision - Arbitrage Reasoning

Manager has two options:

- **Invest** one unit of capital and get returns given: Marginal Product of Capital + Capital Gains - Depreciation Cost
- **Not Invest** and get returns on deposits in the bank.

$$\underbrace{rP_1^K}_{\text{Net Return from Bank}} = MPK + \underbrace{(P_2^K - P_1^K)}_{\text{Capital Gains}} - \underbrace{\delta P_2^K}_{\text{Depreciation Costs}} \tag{11}$$

- For **profit maximization**, both activities should yield the **same return** by no arbitrage condition.
- **How?** If return on K dominates, invest more in K ($\max \pi$), MPK falls, closing the gap (and vice versa)

Investment Decision - Arbitrage with Taxes on Profits

- Aritrage arguments can also be used as a way to easily understand the impact of **policies on investment**.
- Example: How does an increase in the **corporate tax rate on profits** impact investment?
- An increase in tax rate means that the firm has to pay an additional amount as **taxes from its marginal product of capital.**

With tax rate τ:

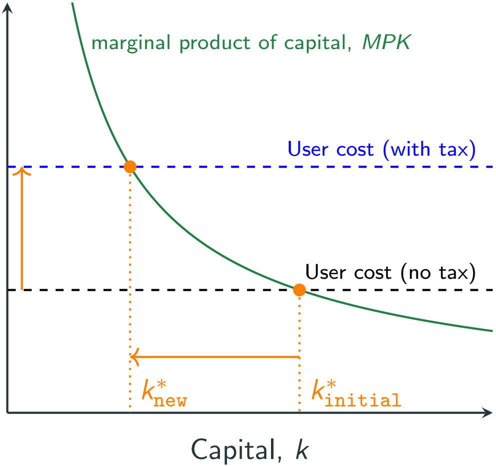
$$rP_1^K = (1-\tau)MPK + (P_2^K - P_1^K) - \delta P_2^K$$

Rearranged:

$$MPK = \frac{rP_1^K - (P_2^K - P_1^K) + \delta P_2^K}{1-\tau}; \quad \tau \in [0,1]$$

Production function and marginal product

$$MPK = \frac{rP_1^K - (P_2^K - P_1^K) + \delta P_2^K}{1 - \tau}$$



2 Period Investment Model - Pros

- ✔ **Simplicity** very simple in set-up and optimality
- ✔ **Transparent Rule**: optimal capital: equate MPK and user cost
- ✔ **Immediate Policy Relevance** shows how higher interest rates/monetary policy, depreciation allowances, taxes on profits will influence investment

2 Period Investment Model - Critiques

- ✖ **Predicts Jump adjustment:** predicts K will adjust very quickly and completely whenever parameters change

- ✖ **Var(Investment) cannot be explained–** variance of (P_t^K, δ_t, r_t) can't explain variance of Inv in data

- ✖ **No bounds on investment rates** - no cost of expanding capital, then even infinite *rates* of investments $(\mathcal{I}_t/K_t)$ are theoretically possible!

(!) **Constant returns to scale** a.k.a. $\pi(k) = Ak$ models are bad news (why? what have we assumed in the plot?)

2 Period Investment Model - Critiques

- **Slow, hump-shaped adjustment** Real firms take action over several periods in hump-shaped responses.
- **Growing pains / Adjustment is costly**: a large investment (e.g. 50% of capital) is not just a small one (1%) rescaled when it comes to installation and operations
- **Expectations of the future matter** forward-looking: investment decisions are also based on expectations of future profits (productivity, demand, cost, opportunities).
- Motivates **Tobin's Q model, or the Adjustment Cost model**.