

---

---

Tutorial 2: Investment and Lagrangians  
Investment, Finance and Asset Prices; ECON 5068

---

---

## Firm's Problem - Lagrange Multiplier

Here is a variation of the firm's problem we studied in class. Output is given as a Cobb-Douglas production function,

$$Y_t = A_t K_t^\alpha L_t^\beta \quad \alpha, \beta > 0, \quad \alpha + \beta \leq 1$$

where,  $K_t$ ,  $L_t$  denote capital and labor respectively and  $A_t$  denotes the level of productivity. The firm's (static) profit is given as,

$$\Pi_t = Y_t - w_t L_t$$

where  $w_t$  denotes the wage rate. The firm chooses  $L_t$  to maximize  $\Pi_t$  period by period.

- a. Perform the maximization of profits and derive the amount of labor the firm would like to hire optimally. Once you get this, substitute into the profit function and derive profits as a function of  $K_t$ ,  $A_t$  and  $w_t$  only. To help out you might want to use the result that with a Cobb-Douglas production labor costs ( $w_t L_t$ ) are proportional to output.
- b. Once you have the expression for profits from above, set up the Lagrangian of the following problem to choose optimal investment.

$$\max E_0 \sum_{t=0}^{\infty} \left( \frac{1}{R} \right)^t Div_t$$

such that

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{1}$$

$$AC_t = \frac{\phi}{2} \left( \frac{I_t}{K_t} \right)^2 K_t \tag{2}$$

where  $\delta$  is the depreciation rate of capital,  $Div_t$  is dividends at time  $t$  and  $AC_t$  is adjustment costs of capital at time  $t$ . First, define dividends, then set up the Lagrangian and perform the maximization with respect to the relevant choice variables. From your first order conditions derive the optimal investment rule.