ECON5068: Investment, Finance and Asset Prices

Solution Sheet

Practice Exams 2024 & 2023

2024 Examination Solutions

Q1.1: Question 1: Uncertainty and Investment

1.1 Real Options and Investment Under Uncertainty [20%]

Increased uncertainty affects firm investment decisions significantly when investment is partially irreversible. Under (partial) irreversibility, firms cannot costlessly reverse capital investments. This can be because secondary markets markets offer lower prices than purchase prices (the Wedding Dress effect), or capital is firm-specific with limited alternative uses (Toyota can't really sell their assets to Honda, the city of Glasgow can't sell its canals to Edinburgh, they are location-specific), the extraction process could break the capital, or it is simply too destructive, costly, or dangerous (Nuclear reactor)

Real Options Theory says that investment opportunities represent call options on future profits – the right but not obligation to invest. Like financial options, these have time value: waiting to invest preserves the option to learn about future business conditions like demand, costs, or technology. We saw in class how option value of waiting is related to the expected regret we feel in the cases of losses. The option is priced as the average we would be willing to pay to reverse a mistake in the next period. Think of the Windows versus Mac argument. If I am locked into Mac ($M > W$), the only time I feel regret is when my ranking $M' < W'$ is reversed next period. If $M' > W'$ Mac is still the best choice then it doesn't affect my choice, only confirms it.

$$OptionValue = Z = \beta E(regret = V_{t+1}(j) - V_{t+1}(i) | Bad\ news\ for\ choice\ i\ when\ I\ have\ locked\ into\ i)$$

Higher uncertainty increases this option value because it raises the chance the options rankings are changed, making delay more attractive. Even if all plan/project returns rise, if the dispersion of expected future returns increases, this raises the chances projects will reorder their rank of profitability, and so we would regret out choice if we committed to any specific plan –

this in Bernanke's Bad News principle, only the probability and cost of bad news matters for committing to the investment, not upside risk.

Higher uncertainty, higher option value, better to wait and see and remain uncommitted until uncertainty resolves: wait-and-see

In times of high uncertainty (policy uncertainty, demand volatility, financial crises, Brexit, Trump Trade Wars), firms might adopt a "wait-and-see" approach. We don't know which specific project will be best in the future economic environment, and if we lock in now there is a high chance that ranking will change. Investment becomes less responsive to fundamentals like Tobin's Q or cash flow. Empirically, this shows up as investment declining and becoming less sensitive to profitability measures during high-uncertainty episodes (e.g., Brexit, COVID-19, elections).

1.2 Policy Design Under Uncertainty [15%]

When investment actions are irreversible, uncertainty is the responsiveness killer. Policymakers must take into account investment sensitivity to business conditions and the dampening effects of uncertainty – even great policy that improves returns for firms a lot might not work if the firm is in wait-and-see mode due to precautionary behaviour.

Traditional policies—investment tax credits, accelerated depreciation, or interest rate cuts—improve expected returns to capital. However, these may be of little impact during high uncertainty periods like recessions. In times of high uncertainty, the option value of waiting can dominate improved returns from stimulus policy.

Stimulus policies are usually just temporary, trying to create urgency to invest in the present period. But if fundamental uncertainty (about macro conditions broadly: demand, technology, regulations) is still high for a long time, firms may optimally forgo temporary incentives rather than commit irreversibly under uncertainty. A few periods from now the stimulus is gone, but we are stuck regretting our mistakes if things go bad. Who is going to buy my tailor-made suit, even if the price is subsidised?

The theory of irreversible investments due to price wedges, capital destruction, specificity (etc.) predicts we should:

- **Forward Guidance** Provide clear, credible policy commitments;
- **No flip-flops** avoid policy reversals and sharp changes of direction where possible. UK Governemnt has been very flip-floppy for a long time...

- **Introduce flexibility** Subsidize adjustment costs; improve resale markets for capital, try to improve flexibility of new investments, so firms can pivot if conditions change. Some technologies have uses across a broad range of places and sectors, others are completely stranded. Good policy will respect this.
- **Durable Committent** Policy should be long-term in duration, so firms know the setting will not change radically from now until then, there is no uncerainty to resolve by waiting.
- **Irreversible Investments rank last** Stimulus should put more importance on projects which do not suffer from the consequences of irreversibility or specificity: a recession is the wrong time to evaluate the choice between a new hydroelectric dam versus solar farm.

Reducing uncertainty for firms may be just as important as conventional demand-targeted policies in recessions when uncertainty is high and thus responsiveness to conditions is low.

1.3 Empirical Testing Strategy [15%]

Baseline Specification:

$$\frac{I_{it}}{K_{it-1}} = \alpha_i + \gamma_t + \beta_1 Q_{it-1} + \beta_2 \sigma_{it} + \beta_3 (Q_{it-1} \times \sigma_{it}) + \beta_4 \frac{CF_{it}}{K_{it-1}} + \mathbf{X}'_{it} \boldsymbol{\delta} + \varepsilon_{it}$$

where I_{it}/K_{it-1} is the investment rate, Q_{it-1} is Tobin's Q (or sales growth as proxy), σ_{it} is firm-level uncertainty (stock return volatility, sales volatility, or text-based measures), CF_{it} is cash flow, and $\mathbf{X}_{it}$ includes controls (leverage, size, age). Firm fixed effects α_i control for time-invariant heterogeneity; time effects γ_t absorb aggregate shocks. Controls $\mathbf{X}_{it}$ make sure we are comparing apples with apples: large firms with other large firms etc.

Parameter of Interest: β_3, the interaction term is the key parameter. Assume β_1 is positive, so investment is sensitive to Q and profitability conditions. Higher-than-usual uncertainty can affect investment directly ($\beta_2 < 0$), meaning no matter what fundamentals are, uncertainty hurts investment levels; but ALSO that is reduces responsiveness of investment to Q: so we can write sensitivity of investment to Q as:

$$\frac{\partial (I/K)_{it}}{\partial Q_{it}}(\sigma_{it}) = \beta_1 + \beta_3 \sigma_{it}$$

Hopefully this specification makes clear we allow investment-sensitivity to depend on uncertainty. A negative coefficient (parameter estimate) for β_3 means uncertaint reduces responsiveness to Q.

Identification Concerns: Uncertainty may be endogenous (unknwons inside ε_{it} affect both uncertainty and investments, poor performing firms are likely to be suffering negative productivity and/or demand shocks). Address via:

- *Instrumental variables:* Use industry-level or macro uncertainty as instruments
- *Quasi-experiments:* Exploit sudden uncertainty spikes (Brexit, elections, policy changes) as exogenous shocks

Expected Results: The interaction term β_3 should be negative and significant, indicating that high-uncertainty firms are less responsive to investment opportunities. Effects should be stronger for:

- Firms with irreversible capital (manufacturing vs services vs construction vs retail)
- Financially constrained firms (limited ability to wait, we need to grow to survive)
- During aggregate uncertainty episodes

Q1.2: Question 2: Investment and External Finance

2.1 Bellman Equation and Optimal Investment [25%]

The firm maximizes the present discounted value of dividends. Under the assumption $I_t > \theta_t K_t^\alpha$, dividends equal:

$$\begin{aligned} D_t &= \theta_t K_t^\alpha - I_t - \frac{\phi}{2} I_t^2 - \mu(I_t - \theta_t K_t^\alpha) \\ &= (1+\mu)\theta_t K_t^\alpha - (1+\mu) I_t - \frac{\phi}{2} I_t^2 \end{aligned}$$

The $(1+\mu)$ multiplier on profits reflects that internal funds are more valuable than external funds—each dollar of profit saves μ in external financing costs.

Bellman Equation:

$$V(K,\theta) = \max_{I,K'} \left\{ (1+\mu)\theta K^\alpha - (1+\mu) I - \frac{\phi}{2} I^2 + \beta E_{\theta'|\theta}[V(K',\theta')] \right\}$$

subject to

$$K' = (1-\delta)K + I$$

.

$$\boxed{\mathcal{L}(K,\theta,\lambda) = (1+\mu)\theta K^\alpha - (1+\mu) I - \frac{\phi}{2} I^2 + \beta E_{\theta'|\theta}[V(K',\theta')] - \lambda(K' - I - (1-\delta)K)}$$

First-Order Conditions with the help of envelope condition and stationarity of value function:

$$\begin{aligned}
[I:] &\quad -(1+\mu) - \phi I + \lambda = 0 && (1)\\
[K':] &\quad \beta \frac{\partial EV(\theta', K')}{\partial K'} - \lambda = 0 && (2)\\
[EC:] &\quad \frac{\partial V(\theta, K)}{\partial K} = \frac{\partial \mathcal{L}}{\partial K} = (1+\mu)\alpha\theta K^{\alpha-1} + (1-\delta)\lambda && (3)\\
[EC+ST:] &\quad (1+\mu)MPK' - (1-\delta)\lambda' && (4)
\end{aligned}$$

Combining the pieces:

$$1 + \mu + \phi I = q \tag{5}$$

$$1 + \mu + \phi I = \beta E[V_{K'}(K', \theta')] \tag{6}$$

$$1 + \mu + \phi I = (1+\mu)MPK' + (1-\delta)q' \tag{7}$$

Marginal cost: price of the capital ($p = 1$) plus the cost of external finance per unit investment μ plus marginal pain of adjustment ϕI. Costly finance increases marginal costs.

Marginal benefits: extra capital means extra production and profits. Profits are valuable as dividends but also save external finance costs. The more we can pay with own funds the less we pay the bank in costs, this is a benefit, so we value MPK' times $(1+\mu)$, finally we have the usual residual value of capital priced as shadow/internal price q, the lagrange multiplier.

Optimal Investment Condition: Combining the FOC and envelope condition:

$$I_t = \frac{1}{\phi}\left[q_t - (1+\mu)\right]$$

In corporate finance lingo, q_t is the expected return on investment, while $(1+\mu)$ would be the hurdle rate the project needs to clear in order to be chosen.

2.2 Marginal Q [10%]

Based on the above derivations and discussion, marginal Q is defined as:

$$q = \beta EV_{K'}(\theta', K') = (1 + \mu + \phi I) \tag{8}$$

2.3 Effects of External Financing Costs [15%]

Direct Effect on Investment Decision: External financing costs μ act like a tax on investment, raising the hurdle rate. This wedge reduces optimal investment relative to the frictionless case where $\mu = 0$. The adjustment cost parameter ϕ determines how sensitive investment is to changes in q.

$$I_t = \frac{1}{\phi}\left[q_t - (1+\mu)\right]$$

Secondary 'Financial Accelerator' effect: NB the firm value will be reduced by the presence of external finance costs $\mu > 0$. From envelope condition we can evaluate how firm value moves with the external finance premium:

$$\frac{\partial V(\theta, K)}{\partial \mu} = \frac{\partial \mathcal{L}}{\partial \mu} = \theta K^{\alpha} - I$$

So the change in value depends on whether the firm is a net-saver or net-borrower. In our case we assume borrowing by construction:

$$\frac{\partial V(\theta, K)}{\partial \mu} < 0 \quad if \quad \theta K^{\alpha} < I \tag{9}$$

This establishes one of the links in the Financial Accelerator mechanism: as financial costs rise, firm value falls, the next link would be lenders dislike falling firm value, so tighten lending conditions further (higher μ or a lower borrowing limit not modelled explicitly)

3.1: Investment–cash flow sensitivity is a reliable indicator of financing constraints at the firm level."

Recall that under Q-theory, the variable marginal Q should contain all relevant information for the firm's investment decision - it is a sufficient statistic in Tobin's world. If we know Q that is enough, so a test of Q-theory comes in the form of a regression of investment rate (I/K) on Q and a measure of cashflows.

In the baseline Tobin model we don't make any deeper assumptions about the financial structure of the economy – the firm will always invest if it is profitable to do so. So when capex exceed revenues, the firm is able to borrow with interest rate $r = 1/\beta - 1$, recall the firm can move financial flows forward or backward in time using discount rate $\beta = 1/(1+r)$. If we assume

$\beta = 0.96$ this is an interest rate of about $r =$4 percent.

But we know financial markets are far from perfect. There are also sorts of financial positions I should be able to trade but cannot (the instrument doesn't exist, or the price to buy such a position is prohibitive). Young firms have no established credit record, small firms do not have much capital assets to pledge as collateral against the value of credit. Hence we can imagine it is hard for these firms to borrow, and so the assumption of saving/borrowing at rate $r = 0.04$ is unrealistic.

FHP run regressions of the form

$$(I/K)_{jt} = \alpha_j + \beta_1 \cdot averageQ_{jt-1} + \beta_2 \cdot (CF/K)_{jt} + controls_{jt} + error_{jt}$$

using panel data of $j = 1, 2...N$ firms for $t = 1, 2, ...T$ years. They estimate the parameters $(\alpha_j, \beta_1, \beta_2)$ using Least Squares (minimising the error between fitted function and the data to the square). Estimated numbers have the hats above their symbols $(\hat{\alpha}_j, \hat{\beta}_1, \hat{\beta}_2)$. marginal Q from the model is not directly observable in data, so FHP use the stock market capitalisation-based average Q as a proxy that captures marginal Q under some special circumstances.

Estimated $\hat{\beta}_2$ is usually high and statistically significant, especially for the FHP group of low-payout firms they take to be constrained, suggesting the cashflow is an important factor in the investment decision according to FHP, which rejects the pure Q theory.

Problems with FHP:

- Proxy variable is not very good: conditions described by Hayashi for CRS are probably not right
- How is Average Q measured: stocks. Very noisy and volatile.
- Stocks might not price assets of new, young, small firms well if not many people know about them
- Who is constrained is open for debate, payout rate might not be great

Improvements with GH:

- GH construct a better proxy called Fundamental Q
- estimates for β_1 on FQ are not so large and imply more realistic investment responses over time

- Cashflow is still important

Other

- One can get significant cashflow coefficients even in a model without frictions, so caution is advised
- the DYNAMIC patterns of cashflow suggest firms increase cash before big lumpy adjustments, then run down with the spike, so looking at a specific moment in time is not fully informative. The response before, during, and after the investment burst contains more information.

Overall the story appears to be that while each study has its limitations the totality of evidence suggests that for certain groups constraints on access to finance are real and important drivers of investment.

3.2 design an empirical strategy to identify the causal effect of financial constraints on investment.

We have not covered this much this year, so it won't be on this years exam, but one of my favourite papers in this area is "The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008–9 Financial Crisis" by Gabriel Chodorow-Reich. How do we distinguish between credit supply shocks from banks/lenders versus changes in financing conditions due to changes in firm fundamentals?

- **Idea behind identification**: Uses dispersion in **lender health** following the Lehman crisis as a source of exogenous (as-good-as-random) variation in credit availability to borrowers, exploiting the fact that firms had **pre-existing banking relationships established before the crisis** that would be costly to switch.
- **Banks often syndicate loans**: they team up. Some banks teamed up with Lehman Brothers (seen as safe before the crash, them they failed). Exposure to Lehman Brothers through the fraction of the bank's syndication portfolio in which Lehman had a lead role.
- So we have **variation on the credit supply** (lender side) which is (arguably) as good as random. A natural experiment. What happens to the firms borrowing from these shocked banks?

- GCR: *I find that credit matters. Firms that had precrisis relationships with less healthy lenders had a lower likelihood of obtaining a loan following the Lehman bankruptcy, paid a higher interest rate if they did borrow, and reduced employment by more compared to precrisis clients of healthier lenders*

2023 Examination Solutions

Question 1: Q Theory (Standard Model)

1.1 Bellman Equation and Optimal Investment [20%]

We have basically already solved this above, in the group assignment, in class etc, so I will be brief here.

Bellman Equation and constraint

$$V(K,\theta) = \max_{K',\mathcal{I}} \left\{ \theta K^{\alpha} - \mathcal{I} - \frac{\gamma}{2} I^2 + \beta E_{\theta'}[V(K',\theta')] \right\} \quad s.t. \quad K' = (1-\delta)K + \mathcal{I}$$

Lagrangian

$$\mathcal{L}(\theta,K,\lambda) = \theta K^{\alpha} - \mathcal{I} - \frac{\gamma}{2}\mathcal{I}^2 + \beta E_{z'|z} V(K',\theta') - \lambda \Big(K' - \mathcal{I} - (1-\delta)k \Big) \tag{10}$$

FOCS

$$\begin{aligned}
[I:] &\quad -(1+\gamma I) + \beta E[V_{K'}(K',\theta')] = 0 && (11)\\
[K':] &\quad \beta \frac{\partial EV(\theta',K')}{\partial K'} - \lambda = 0 && (12)\\
[EC:] &\quad \frac{\partial V(\theta,K)}{\partial K} = \frac{\partial \mathcal{L}}{\partial K} = \alpha\theta K^{\alpha-1} + (1-\delta)\lambda && (13)\\
[EC+ST:] &\quad MPK' - (1-\delta)\lambda' && (14)
\end{aligned}$$

Optimal Investment Rule:

$$I_t = \frac{1}{\gamma}\left[\beta E[V_K(K_{t+1},\theta_{t+1})] - 1\right] = \frac{q-1}{\gamma}$$

Interpretation: The firm equates marginal cost of investment with marginal benefit. Marginal cost includes the purchase price (1) plus convex adjustment costs (γI_t). Marginal benefit is the expected shadow value of capital next period, $\beta E[V_K(K_{t+1},\theta_{t+1})]$.

The adjustment cost parameter γ determines investment sensitivity: higher γ means more sluggish investment response to changes in the shadow value of capital. Investment is forward-looking, depending on expected future returns.

Question 2: Testing Q theory

3.1 The irrelevance of financial conditions on investments is true only when you have perfect capital markets where all firms have easy access to credit at interest rate r. Imperfections in capital markets such as asymmetric information lead to external funds being more expensive than internal funds. You are expected to touch on all aspects of such imperfections and what this means for firm investing. You need to discuss relevant portions from the FHP and the Gilchrist and Himmelberg articles.

3.2 Describe how you would test Q theory and the role of financial constraints using panel regressions.

- **measurement issues** related to Q, stock volatility, learning less about small firms, cashflow may contain extra information not in firm valuations
- the concept of **average and marginal Q**, probably not a good substitution
- how firms can be **classified into financially constrained** and unconstrained firms (slightly ad hoc), reasons why 'constrained firms' might be fundamentally different: smaller, faster decision making, less public information
- the **robustness of this classification**, endogeneity concerns (omitted variables bias) etc

. Importantly, the approach should be connected to what is followed in the Fazzari, Hubbard and Petersen (1988) and Gilchrist and Himmelberg (1995) articles.

Question 3: Brand and Physical Capital

3.1 Labour Law of Motion, Dividends, and Firm Value [15%]

Labour Law of Motion:

Labour evolves according to:

$$L_{t+1} = (1 - \mu)L_t + H_t$$

where H_t is hiring and μ is the exogenous attrition rate. This parallels the capital accumulation equations but for Labour.

Dividends:

Dividends equal operating profits minus all investment costs and wage payments:

$$D_t = \pi(K_t, B_t, L_t) - I_t^K - I_t^B - W_t H_t - C(K_t, B_t, I_t^K, I_t^B)$$

The firm pays:

- Physical capital investment, capeX: $I_t^K = X_t$
- Brand capital investment, marketing: $I_t^B = M_t$
- Wages to hired workers: $W_t L_t$
- Adjustment costs: $C(K_t, B_t, I_t^K, I_t^B)$, here the pain of growing or shrinking the asset base in either dimension is free to depend on the stocks of both variables as well as the size of adjstments. This means investment in one factor may encourage or discourage investment in the other, depending on whether costs are compliments (one boost the cost of the other) or substitutes (investing in one easy the path for the other). Hence inside C we determine if there are congestin effects of synergy effects between X and M

Firm Value:

$$V(K, B, L) = \max_{I_B, I_K, K, B, L} \left\{\pi(K, B, L) - X - M - wL - C(K, B, X, M) + \beta EV(K', B', L')\right\} \tag{15}$$

$$s.t. \quad K' = (1 - \delta_K)K + X \tag{16}$$

$$s.t. \quad B' = (1 - \delta_B)B + M \tag{17}$$

$$s.t. \quad L' = (1 - \mu)L + H \tag{18}$$

Firm Lagrangian:

$$\begin{aligned}
\mathcal{L}(K, B, L, \mathbf{q}) = \; & \pi(K, B, L) - X - M - wL - C(K, B, I_K, I_B) \\
& + \beta EV(K', B', L') \\
& - q_k(K' - (1 - \delta_K)K - X) \\
& - q_b(B' - (1 - \delta_B)B - M) \\
& - q_\ell(L' - (1 - \mu)L - H)
\end{aligned}$$

Or plugging all the constraints in:

$$\mathcal{L}(K,B,L,\mathbf{q}) = \pi(K,B,L) - X - M - wL - C(K,B,X,M)$$
$$+ \beta EV\Big(K(1-\delta_K) + X, B(1-\delta_B) + M, L(1-\mu) + H\Big)$$

FOCs should be familiar is spirit if not form – we set marginal cost equal to marginal benefit. This involves the prices of marketing and capex p_j, plus the marginal adjustment cost C_j

$$[X:]\ q_k = (p_x + C_X) = \beta EV_{K'}(K',B',L')$$
$$[M:]\ q_b = (p_m + C_M) = \beta EV_{B'}(K',B',L')$$
$$[H:]\ q_\ell = w = \beta EV_{L'}(K',B',L')$$

The envelope conditions here are:

$$V_L = \pi_L - w + (1-\mu)q_\ell$$
$$V_K = \pi_K - C_K + (1-\delta_k)q_k$$
$$V_B = \pi_B - C_B + (1-\delta_b)q_b$$

Q-equations, using stationarity and the marginal cost forms of q_j

$$[X:]\quad p_k + C_X(K,B,X,M) = \beta E\left[\pi_K(K',B',L') - C_K(K',B',X',M') + (1-\delta_K)(p_k + C_X(K',B',X',M'))\right]$$
$$[M:]\quad p_m + C_M(K,B,X,M) = \beta E\left[\pi_B(K',B',L') - C_B(K',B',X',M') + (1-\delta_B)(p_m + C_M(K',B',X',M')\right.$$
$$[L:]\quad w = \beta E\left[\pi_L(K',B',L') + (1-\mu)w'\right] \Rightarrow w = \frac{\beta}{1-\beta(1-\mu)}E[\pi_L(K',B',L')]$$
$$q_k = p_k + C_X(K,B,X,M)$$
$$q_b = p_b + C_M(K,B,X,M)$$

With one capital type, investment depends on marginal Q. Here, the firm solves a **portfolio problem**: allocate resources between physical capital K (tangible assets) and brand capital B (intangible assets) and labour force L based on their relative marginal products, depreciation rates (or separation rate in the case of labour), and adjustment costs of each factor.

This is a sufficient stopping point for the exam. For your own knowledge beyond the exam:

Suppose we have a cost function of the form quadratic-additive in X, M terms plus an interaction to determine if we want to do capex simaltaneously with marketing or stagger them. This is control by Greek letter ψ(psi).

$$C(K, B, X, M) = \frac{\gamma_K}{2} X^2 + \frac{\gamma_B}{2} M^2 + \psi \cdot (X \cdot M)$$

Then:

$$C_X = \gamma_K X + \psi M$$
$$C_M = \gamma_B M + \psi X$$

From the FOC we can isolate the $Q_j - 1$ terms

$$1 + \gamma_K X + \psi M = Q_K$$

$$\gamma_K X + \psi M = Q_K - 1$$

From the marketing FOC:

$$\psi X + \gamma_B M = Q_B - 1$$

This is a 2×2 system of equations in (X, M)' and (Q_K, Q_B) and parameters. Solving:

$$\begin{bmatrix} X \\ M \end{bmatrix} = \begin{bmatrix} \gamma_K & \psi \\ \psi & \gamma_B \end{bmatrix}^{-1} \begin{bmatrix} Q_K - 1 \\ Q_B - 1 \end{bmatrix}$$

Let $\Delta = \gamma_K \gamma_B - \psi^2$ (determinant). Then:

$$X^* = \frac{1}{\Delta} \left[\gamma_B (Q_K - 1) - \psi (Q_B - 1) \right]$$

$$M^* = \frac{1}{\Delta} \left[\gamma_K (Q_B - 1) - \psi (Q_K - 1) \right]$$

Verify yourself that is there is no relation in the cost structure between X, M then when $\psi = 0$ we revert to the simpler Tobin model we have seen in class.

The 3 cases:

- **Total separation (boring!):** If $\psi = 0$ (no interaction): $X^* = \frac{Q_K - 1}{\gamma_K}$ and $M^* = \frac{Q_B - 1}{\gamma_B}$. Standard Q-theory. We can examine K, B independent of the other.
- **Congestion between X and M** If $\psi > 0$ (complements in adjustment costs: adjusting one boosts the marginal cost of the other): High Q_B *reduces* capex X^* because it's costly to adjust both simultaneously
- "*Investing in both physical and brand capital simultaneously creates congestion in managerial attention, installation capacity, or organizational bandwidth. There is no capacity for multitasking, we can only do one thing well at a time.*"
- **Synergies between X and M** If $\psi < 0$ (cost substitutes, adjusting one factor reduces the marginal cost of the other): High Q_B *increases* capex X^* because adjusting one makes adjusting the other cheaper.
- *"Joint investment creates synergies based on joint marketing-production coordination and development; suggeting a holistic approach to product creation and marketing; or economies of scope in project management: meaning there is a certain duality to the business. If managers are better informed about production (marketing) they understand the other marketing (production) side better as well."*
- unless there are labour adjustment costs of fluctuating wages, we can solve directly for L^* as a function of state variables and parameters directly.