Decisions under Uncertainty

This tutorial deals with decision making under uncertainty. How do households change consumption preferences when faced with uncertainty? How do firm managers decide on investment projects when cash flow is risky?
You may want to quickly read the following before you solve these questions:

- *Intermediate Microeconomics : A Modern Approach* - Chapter 12 on Uncertainty and from chapter 10, sections 10.6-10.8 on present value.

1 Expected Utility and Uncertainty

The John's (VNM) utility function for money is $u(x) = \sqrt{x}$. John is endowed with a wealth of 9\$. He also holds a lottery ticket that pays 16\$ with probability $\frac{1}{4}$ and pays nothing with probability 3/4.

- What is John's expected utility?
- Find the lowest price at which John would sell the lottery ticket?
- If John's absolute aversion to risk is measured by the Arrow-Pratt quantity:

$$r_A = -\frac{u''(x)}{u'(x)}$$

does John have increasing, decreasing or constant absolute risk aversion?

2 Investment Under Uncertainty

IBM's financial manager is presenting to the board of directors a new investment project. This project involves constructing a production plant in Shenzhen, China. This plant is expected to manafacture a new product which could turn out to be a big success in the chinese market. The manager claims that this production plant will cost 20 million dollars and the construction will last 1 year. The plant is expected to yield cash flows from the second year onwards until the end of time.

Cash flows are dependent on the interest this product receives in the chinese market. In the second year, the new plant will generate 1 million dollars if interest is low; 2 million dollars if interest is moderate; and 3 million dollars if interest is high.
All three events are equally likely. Cash flows from the third year are conditional on second period interest realizations. If interest was initially low then cash flows will be 0.5 million dollars from third year and forever, if interest was moderate then cash flows will be 2 million dollars from third year and forever, and if interest was high then cash flows will be 5 million dollars from third year and forever.
If the time discount rate is $r = 10\%$, then answer the following questions:

- Based on the above information, what is the optimal decision of the board of directors? Do they accept or reject the project?
- Assume that the finanical manager underestimated the cost of new plant by 20%. Will this change your answer to I?
- Consider that from third year and forever, projected cash flows will be 3.5 million dollars instead of 5 million dollars following high market interest in the second year. How will this change your answer to I?