Lab 4: Infinite Horizon Cake-Eating and VFI Convergence
Investment, Finance and Asset Prices; ECON 5068

Question 1: The Infinite Horizon Cake-Eating Problem

1. Solve the inifnite horizon problem using Value Function Iteration.

   Keep in mind two things

   - We know that consumption will be a small share of remaining wealth, so inaccuracies can occur at the lower end of the grid – we will put more grid-points here to boost acccuracy.
   - We know from last weeks Tutorial that the method of undetermined coefficients can give us an analytical solution for the exact fixed point we are solving for.
     - $u(c) = log(c) : V(W) = A + B \cdot \ln(W)$
     - $u(c) = \frac{c^{1-\sigma}}{1-\sigma} : V(W) = \frac{B \cdot W^{1-\sigma}}{1-\sigma}$

1.1 We will solve the first few iterations by hand. Substitute in V_1 that $V_0 = 0$. We start with the fully myopic solution. Living like there is no tomorrow

$$V_1(W) = \max_c [u(c) + \beta V_0(W - c)] = \max_c [u(c)]$$

The second iteration is then

$$V_2(W) = \max_c [u(c) + \beta V_1(W - c)] = \max_c [u(c) + \beta u(W - c)]$$

In general what can we say about the relationship between each iteration in the infinite horizon problem, and the finite horizon solution?

1.2 Set `maxiter=100`; $u(c) = c^{0.5}$; β=0.95; use a non-linearly spaced grid: `wgrid = exp(linspace(log(0.0001), log(1), Nw));` with as close to 1000 points as you can. $W \in (0, 1]$

- Solve for optimal consumption, and store $C_n(W), V_n(W)$ for each iteration $n = 1, 2, ..., 100$.
- Plot for iterations $n = 1, 10, 20, 50$ as well as the fixed point $V^*(W) = BW^{0.5}, B = 1/(1 - \beta^2)^{0.5}$;
- How can we see graphically that the value function $V_n(W)$ is converging to the theorectical object $V^*(W)$
- The value function in the simple cake eating problem can be derived by hand, but in general for more complex models this is not the case. However, hopefully this convinces you that a solution using VFI can still be found, even if with pen and paper it cannot.