Group Assignment

ECON 5068 – Investment, Finance & Asset Pricing, 2025

Due Date: 11 November 2025, 12:00 Midday.

Guidelines

1. You should include your Matlab code in the Appendix. The code should be commented (explain what motivates each block, but no need to go line by line).
2. Figures should be suitably labeled and titled. You can have them either in the main body or in the appendix.
3. Follow standard guidelines for referencing, there should be a bibliography section listing all the references used.
4. You are strongly encouraged to use Overleaf for mathematical formatting. This is an online platform which relies on the coding language Latex for writing scientific documents. You will find the starting preamble code at the end of this document.
5. Some questions have word limits, you should strictly adhere to these limits.

Read the group coursework briefing for further information.

You must answer TWO questions: only one question out of Q1A tobin /Q1B Crusoe and Q2

{1A Tobin -OR- 1B Crusoe} PLUS {Q2 Newton}.

Q1A: Tobin

Consider the problem of a value maximizing firm whose operating profit function at time t is given by

$$\pi(z_t, K_t) = z_t K_t^{\alpha}$$

where z is firm-specific productivity, Which belongs to the grid $\mathbf{z}' = [0.6, 1.0, 1.4]$ and follows an autoregressive process with transition matrix, P

$$P(z_{t+1}|z_t) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \tag{1}$$

K_t denotes capital and α is a parameter representing the elasticity of operating profit with respect to capital. The law of motion of capital is given as

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is investment in capital at time t. Investment is subject to a smooth convex installation cost, expressed as a percentage of capital stock, given by

$$C(I_t, K_t) = \frac{\gamma}{2}\left(\frac{I_t}{K_t}\right)^2 \cdot K_t,$$

where γ is a constant denoting the adjustment cost parameter.

Time is discrete and runs to infinity, $t = 0, 1 \ldots \infty$. Firm manager discounts future values with the factor β.

Based on the above information, answer the following two questions:

(1.1) Write down the Bellman equation and derive the optimal investment decision condition.

Define marginal Q and provide an economic interpretation. How is average Q related to marginal Q for this firm? [20%]

(1.2) Solve the Bellman equation using dynamic programming. A simple grid search is advised. Set model parameters as follows

- $\beta = 0.96$, $\alpha = 0.36$, $\gamma = 0.03$ and $p = 1$. Discretize capital grid with 301 uniformly spaced points in the interval $[0.1, 11]$ using `linspace`.

Plot (i) value function (ii) capital policy (iii) investment policy (iv) dividends (retained earnings). Interpret these graphs.

What are we assuming about financial markets in this model? [Hint: what happens to expenditures and profits at low levels of capital?

Explain how optimal investment responds to

- changes in the adjustment cost parameter.
- changes in how the firm manager values time.
- [BONUS] what if we “turn off" the assumption mentioned above in (1.2)?

Answer in not more than 500 words. [40%]

Q1B: Crusoe

Robinson Crusoe is the sole owner and employee of CrusCorp, based on Castaway Island. He has log utility (not linear!) so his objective is to maximise lifetime utility of consumption, funded by his retained earnings from CrusCorp. He doesn't care about his hours of work (which we ignore in production). The objective he maximises is therefore:

$$E_0 \sum_{t=0}^{\infty} \log(Div_t)$$

CrusCorp produces with capital only, and Crusoe consumes out of the firm's dividends/retained earnings – what is left of production after investment. The law of motion of capital is standard, and there are no capital adjustment costs. The resource constraint is then production = consumption plus investment: $Y = C + I$, or:

$$C_t = Div_t = f(K_t) - [K_{t+1} - (1-\delta)K_t]$$

Remember! Crusoe must always consume:

$$C_t > 0 \quad \forall t$$

The production function at time t is given by

$$f(K_t) = z_t K_t^{\alpha}$$

where z is productivity, which belongs to the grid $[0.6, 1.0, 1.4]$ and follows an autoregressive process with transition matrix

$$P(z_{t+1}|z_t) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \tag{2}$$

Time is discrete and runs forever, $t = 0, 1 \ldots \infty$. Crusoe discounts future values with the factor β and starts with some intial capital $K_0 > 0$

Based on the above information, answer the following questions:

(1.1) Write down the Bellman equation and all necessary constraints. Set out the Recursive Lagrangian for this problem and derive the optimal investment decision condition (note this is enough to pin down consumption). Define intertemporal optimality and provide an economic interpretation. [20%]

(1.2) Solve the Bellman equation using dynamic programming. A simple grid search is advised.
Set model parameters as follows

- $\beta = 0.96$, $\alpha = 0.33$, $\delta = 0.08$, $u(c) = \log(c)$.
- Discretize capital grid with 301 uniformly spaced points in the interval $[0.1, 11]$.
- Hint: You might find it helpful to use (negative) machine infinity `Inf` which matlab will treat as a very large number greater than anything we can specify. This is useful when imposing certain constraints! `-1e10` $= -1 \times 10^{10}$ is an alternative.

Plot (i) value function $V(z, k)$ (ii) consumption policy $c(z, k)$ (iii) capital policy, $k'(z, k)$ (iv) net investment policy $(k' - k)$ functions. Interpret these graphs. Explain how optimal investment responds to

- changes in how Crusoe values time.
- changes in the island's climate so that depreciation is higher

Answer in not more than 500 words. [40%]

Q2: Newton

Consider a two period firm model similar to the one you studied in Unit 1 modified to include uncertain productivity. In period 1, the firm starts with some capital $K_t = K_1$ that it owns. The firm generates operating profits using the function

$$\pi(K_t) = \theta_t K_t^\alpha \qquad\qquad t = 1, 2.$$

where the productivity level θ_t evolves randomly across periods. In each period, productivity can be one of three possible states:

$$\theta \in \{\theta_L, \theta_M, \theta_H\}$$

representing low, medium and high states respectively. The firm starts period 1 with low productivity θ_L. The evolution of productivity follows the transition matrix P :

$$P(\theta_{t+1}|\theta_t) = \begin{bmatrix} 0.60 & 0.25 & 0.15 \\ 0.10 & 0.80 & 0.10 \\ 0.00 & 0.30 & 0.70 \end{bmatrix}$$

The firm decides on investments I_t in each period which becomes productive capital with one period delay. Capital depreciates every period at the rate of δ, and follows the standard law of motion. The firm discounts future values with the discount factor β. The firm is liquidated at the end of period 2 and the revenues from liquidation is distributed as dividends. Assume that all prices are constant and normalized to unity.

Based on this information, answer the following two questions:

2.1 What are the state and choice variables in this model? What is the value of the firm? Write down the sequential constrained optimization problem of the firm and derive first order conditions that characterize optimal investment policy. Interpret these equations. [20%]

2.2 Numerically solve for investments using Newton's method if $K_1 = 5, \beta = 0.96, \alpha = 0.33, \delta = 0.025$; and $\theta \in \{0.7,\ 1.0,\ 1.3\}$ are the low, medium and high state productivities. Plot and interpret policy functions over $K_1 \in [3, 12]$ and $\theta = \theta_L$. [20%]

1 *Overleaf code to get started

```
\documentclass[11pt]{article}
\usepackage[margin=0.7in]{geometry}
\linespread{1.3}
\usepackage{setspace}
\usepackage{amsmath,amssymb}
\usepackage{anyfontsize}
\usepackage{booktabs, multicol, multirow}
\setlength\parindent{pt}
\usepackage{graphicx}
\usepackage{mathrsfs}
\usepackage[T1]{fontenc}
\usepackage{natbib}
\usepackage[capposition=top]{floatrow}
\usepackage{float}
\usepackage{titlesec}
\DeclareMathOperator{\E}{\mathbb{E}}
\usepackage[pagewise]{lineno}
\usepackage[usenames,dvipsnames]{color}
\usepackage[colorlinks=true]{hyperref}
\hypersetup{
    colorlinks = Maroon,
    citecolor  = {Maroon},
    linkcolor  = {Maroon},
    urlcolor   = {Maroon},
    }
\usepackage{comment}
\usepackage{subfigure}
\begin{document}
\hline\hline
%smallskip
\begin{center}
{\Large\texttt{Group Assignment IFAP 2025}}
My Names Here}
\end{center}\\ \\
\hline\hline
\\
\section*{{Question 1}}


\section*{{Question 2}}

\end{document}
```

End of Assignment.