Investment, Finance and Asset Prices

ECON 5068

Practice Exam 2023

Duration: 2 Hours

Instructions:

- Answer **TWO** questions.
- Marks for each part are shown in brackets, 100 marks total.
- Show all your working.
- Refer to Mathematical Appendix if needed

You must answer any TWO questions. All questions are of equal weighting.

Q 1: Q theory [50 marks]

1. Consider the problem of a value maximizing perfectly competitive firm whose profit function at time t is given by $\theta_t K_t^\alpha$ where θ_t denotes the productivity level, K_t denotes capital and α is a parameter representing the elasticity of output with respect to capital. The law of motion for capital is standard and is given by

$$K_{t+1} = (1-\delta)K_t + I_t$$

where I_t is investments in capital at time t and δ is the depreciation rate. The price of a unit of capital good is normalized to 1 and investment is subject to smooth convex installation cost given by

$$C(I_t, K_t) = \frac{\gamma}{2} I_t^2,$$

where $\gamma > 0$ is a constant. Time is discrete and runs to infinity, $t = 0, 1 \ldots \infty$. Future values are discounted with the factor β. The productivity level for the firm is stochastic and follows an independent and identically distributed (i.i.d) Normal process:

$$\theta_t \overset{iid}{\sim} N(0, \sigma^2), \quad t = 0, 1, 2, \ldots \infty$$

Productivity θ is thus assumed to be serially uncorrelated. Based on the above information, answer the following questions:

1.1 Write down the Bellman equation. Derive and interpret the optimal investment decision condition.

[20%]

1.2 Define marginal Q and provide an economic interpretation.

[15%]

1.3 How is average Q related to marginal Q for this firm?

[15%]

Q 2: Q theory [50 marks]

2. Based on the Fazzari, Hubbard and Petersen (1988) article and the investment-external finance literature, answer the following questions:

   1.1 Do you agree with the following statement - "*A firm's financial structure is irrelevant to investments because external funds provide a perfect substitute for internal funds*". Explain in less than 500 words.

   [25%]

   1.2 If you have access to a panel dataset containing firm level variables observed for some periods, describe in not more than 500 words how you would test for the effect of financial constraints on firm investments.

   [25%]

Q 3: Brand and Physical Capital [50 marks]

3. Consider the problem of a firm that invests in intangible capital (brand capital) in addition to physical capital and labor.

   The firm's operating profits at time t is $\pi(K_t, B_t, L_t)$ where K_t, B_t and L_t are physical capital, brand capital and labor at time t, respectively. The firm invests in physical capital I_t^K and brand capital I_t^B every period. The two capital stocks evolve as follows:

$$K_{t+1} = (1 - \delta_K)K_t + I_t^K$$

$$B_{t+1} = (1 - \delta_B)B_t + I_t^B$$

   where δ_K and δ_B are the exogenous depreciation rates of physical capital and brand capital, respectively. The firm hires H_t amount of labor every period. However, a fixed rate of labor, denoted by μ, leave every period for voluntary reasons.

   Firm incurs costs when adjusting stocks of physical and brand capital of the form $C(K_t, B_t, I_t^K, I_t^B)$. This cost function is convex and increasing in investments of both capital units. There is no adjustment cost involved in hiring labor. Labor is paid at the wage rate W_t. The prices of both physical and brand capital are constant and normalized to unity. The firm takes all prices including the wage rate as given.

   Finally, time is discrete and the firm lives forever, $t = 0, 1, 2 \ldots \infty$. The firm discounts time using the constant factor β.

   Based on this information, answer the following questions:

   3.1 Write down the law of motion for labor and state the expression for dividends. What is the value of the firm? [15%]

   3.2 State the optimization problem of the firm in sequential form and solve it using Lagrange Multiplier method to obtain the system of equations that characterize firm's optimal investment policy in all variable inputs. You should provide an interpretation of these equations. [20%]

   3.3 If operating profits net of all costs are taxed at the rate τ, how does investment in physical and brand capital respond to changes in this corporate tax rate? [15%]

END OF EXAMINATION

Mathematical Appendix:

- Power rule:

$$\frac{d}{dx}x^n = nx^{n-1}$$

- Logarithmic rule:

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

- Logarithmic function rule:

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}$$

- Chain rule:

$$\frac{d}{dx}f(g(x)) = f_x(g(x)) \cdot g_x(x)$$

- Product rule:

$$\frac{d}{dx}[u(x)v(x)] = u_x(x)v(x) + u(x)v_x(x)$$

- Geometric Series:

$$S = a + ar + ar^2 + ... = a(1 + r + r^2 + ...) = \frac{a}{1 - r}$$